Sum Rate Maximization Based on Joint Power Control and Beamforming in MIMO One Way AF Relay Networks

Shahriar Shirvani Moghaddam[1*] and Maryam Alibeigi[1]

[1]*Faculty of Electrical and Computer Engineering, Shahid Rajaee Teacher Training University (SRTTU), Lavizan, 1678815811, Tehran, Iran.*

Authors' contributions

This work was carried out in collaboration between two authors SSM and MA. Author SSM designed the study and managed the research procedure. Author MA performed the numerical simulations in MATLAB software and wrote the first draft of the manuscript. Consequently, author SSM checked the analyses and simulation results and also rechecked the first draft of manuscript. Finally, authors SSM and MA read and approved the final manuscript.

Original Research Article

ABSTRACT

Aims: Joint Power control and beamforming in MIMO one way AF relay networks by obtaining beamforming weights on relay nodes through maximizing sum rate while the total power consumed in all relay nodes are not greater than the certain predefined threshold.
Study Design: Comparative study by simulation in MATLAB software.
Place and Duration of Study: Digital Communications Signal Processing Research Lab., Shahid Rajaee Teacher Training University, Since June 2012 to June 2014.
Methodology: We consider a wireless communication network consisting of d source-destination pairs communicating in a pairwise manner via R non regenerative one way relay nodes. Our objective is maximizing sum rate supposing that the total power consumed in all relay nodes is not greater than the certain predefined threshold. It is shown that finding beamforming matrix which satisfies our goal is a non-convex problem. So, we use semidefinite relaxation technique to turn

_____

Corresponding author: E-mail: sh_shirvani@srttu.edu;

this problem into a semidefinite programming (SDP) problem. We propose two new algorithms which maximize total signal to total leakage (interference plus noise) ratio (TSTLR).

Results: Simulation results show that our proposed algorithm is a low-complex algorithm and outperforms the existing total leakage minimization algorithm, named TL. In addition, the effect of the number of relays, the number of transmitter-receiver pairs, quality of uplink as well as downlink channels and imperfect channel state information (CSI) are investigated.

Conclusion: The proposed method maximizes the total signal to total leakage ratio by maximizing the difference between the numerator and the denominator. It is shown that when the number of relays increases or the quality of uplink and downlink channels improves, higher sum rate can be achieved. Moreover, by increasing the number of source – destination pairs the sum rate increases, too. Also, it is shown that when uncertainty of channels increases the achievable sum rate decreases.

Keywords: AF relays; beamforming; power control; sum rate maximization.

1. INTRODUCTION

Due to the increasing requests for wireless communication services, we need systems which support higher data rates, capacity and quality in a joint state. Limited allocated frequency spectrum forces the wireless networks to use improved protocols and algorithms supporting higher spectral efficiency to achieve higher bit rate [1].

Efficient use of bandwidth and two inherent abilities, achieving higher transmission data rates and offering higher bit rates are important advantages of multi input multi output (MIMO) communication systems. On the other hand, the efficiency of these systems will be decreased by undesired phenomena such as noise, interference and fading. Therefore, in order to improve the performance of these systems, several techniques such as power control, beamforming and space-time coding have been proposed and investigated. The most fundamental problem in a large wireless network which shares a limited frequency band is co-channel interference (CCI) and it is inevitable due to frequency reuse [2]. Also, it can reduce system performance. Therefore, some research works are focused on the methods that reduce CCI. Generally, beamforming and power control are used jointly in CCI wireless systems to increase system performance.

However, implementing multiple transmit antennas in mobile terminals may not be always feasible due to power and space limitations as well as computational complexity. One approach to tackle these practical restrictions is to use relays [3] as virtual antennas helping communication between transmitter and receiver. The main idea of these networks is to allow different nodes to be involved in transmission by making virtual array antennas. This method, avoids the problems related to implementing antenna arrays such as, physical limitations, mutual coupling and so on. Moreover, it makes the possibility of taking diversity and other benefits of MIMO systems and also applying ideas introduced for MIMO systems, beamforming and power control.

It should be mentioned that for long distance communication between transmitter and receiver, the direct path experiences large attenuation which greatly reduces the communication quality. Hence, using relay based networks, transceiver nodes can connect to each other in two time steps. In the first step, transmitter sends the signal to relay and relay processes the received signal. This process in relay node depends on the type of relay. Afterwards, in the second step, the relay sends the processed signal to the receiver.

Relays can be classified based on the type of signal processing operation. For this purpose, several schemes have been proposed. Some of them are amplify-and-forward (AF), decode-and-forward (DF), compute-and-forward (CF) and estimate-and-forward (EF). Among these schemes, the most effective and popular one is AF. AF relay applies linear signal processing to the signal and forwards it to the receiver. AF relays are called non regenerative because decoding is not performed on them. Although they have no effect on the decreasing of error in system [4], AF relay offers less complexity and higher processing speed with respect to other schemes.

Several algorithms are introduced and evaluated in joint beamforming and power control for MIMO

relay networks. In [5], joint power control and receiver beamforming in MIMO relay networks are used to reduce CCI. In [6], in order to improve the performance of system, power control and beamforming at both transmitter and receiver are implemented. In usual wireless communications systems just a single source and a single destination communicate with each other through multi relays [7-9]. In the most of investigations, due to the high attenuation, the direct path between the source and the destination is ignored. In contrast, in some studies such as [3], the direct path is also considered but this path is the dominant one. The proposed method in [7] cannot be used when the number of pairs of source - destination is more than one. In [6,10] the consumed power is minimized while the required signal to interference plus noise ratio (SINR) of each path is guaranteed. They used semidefinite relaxation technique to turn the problem into a semidefinite programming (SDP) problem which can be solved by using interior point methods. In [10], pair of source – destination and multiple relays which all nodes have just a single antenna are considered. In contrast, a system has been studied in [6,11] which includes pair of source - destination and multiple relays that all of them are equipped with multiple antennas. For a network with pair of transmitter - receiver with multiple relays considering various receiver filters, the beamforming of the transmitter and relays are optimized in [12]. Ref. [13] proposes the optimal joint source and relay power allocation to maximize the end-to-end achievable sum rate. Transmit pre-coders, relay matrices and receive decoders are optimized in [14] to maximize the achievable sum rate.

In this paper, a network consisting of multi pairs of transmitter - receiver that communicate with each other using multi non regenerative relays is considered. In this investigation, an algorithm is suggested which finds an optimal beamforming of relays and maximizes sum rate subject to relay power constraint.

The rest of the paper is organized as follows. In the Section 2, system model is described. Then, we formulate the end-to-end sum-rate maximization problem and present our proposed approach. Simulation results for the different modes of the system are carried out to evaluate the performance of the proposed algorithm in Section 3. Finally, Section 4 presents

conclusions and introduces some suggestions for further works.

2. SYSTEM MODEL

The system consists of two pairs of source - destination with R relays. It is assumed that there is no direct path between the source and the destination as shown in Fig. 1. Each relay transmits an amplitude- and phase-adjusted version of its received signal. In other words, relay multiplies its received signal by a complex weight and then transmits it to the receiver.

Assuming that the coefficient matrix of the channel between pth source and rth relay and also between rth relay and pth destination are denoted by f_{rp} and g_{rp}, respectively, the received signal at rth relay can be obtained by (1) [10].

$$x_r = \sum_{p=1}^{d} f_{rp} s_p + v_p \qquad (1)$$

where s_p is the information symbol transmitted by the pth source and v_r is the additive zero mean noise at the rth relay node.

Using the following assumptions throughout this section, (1) can be rewritten in the form of (2).

1- The relay noise is white, i.e. $E\{v_r v_{r'}^*\} = \sigma_v^2 \delta_{rr'}$ where σ_v^2 is the relay noise power.
2- The pth source uses its maximum power Pp, i.e., $E\{|s_p|^2\} = P_p$.
3- The information symbols transmitted by different sources are uncorrelated, i.e., $E\{s_p s_q^*\} = P_p \delta_{pq}$.
4- The information symbols and the rth relay noise are statistically independent.

$$\mathbf{X} = \sum_{p=1}^{d} \mathbf{f}_p \, s_p + \mathbf{V} \qquad (2)$$

where $\mathbf{X} = [x_1 \, x_2 \, ... \, x_R]^T$, $\mathbf{V} = [v_1 \, v_2 \, ... \, v_R]^T$ and $\mathbf{f}_p = [f_{1p} \, f_{2p} \, ... \, f_{Rp}]^T$.

The rth relay multiplies its received signal by a complex weight coefficient w_r^*. Thus, signals transmitted by all relays can be expressed as (3).
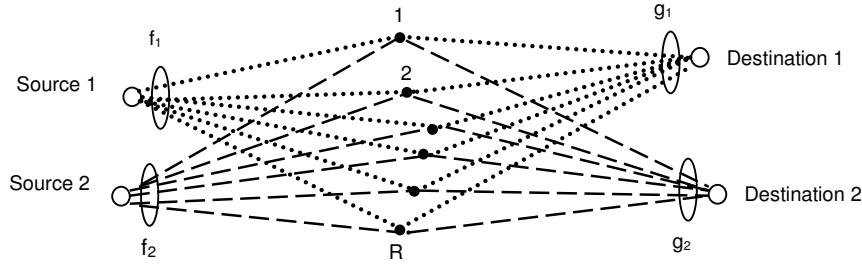
Fig. 1. A system consists of R relays and 2 source-destination pairs

$$\mathbf{t} = \mathbf{W}^H \mathbf{X} \tag{3}$$

where $\mathbf{W} = diag(w_1, w_2, ..., w_R)^T$ and t are R × 1 vectors whose rth entry is the signal transmitted by the rth relay. Denoting the vector of the channel coefficients from the relays to the kth destination as $\mathbf{g}_k = [g_{1k}\ g_{2k}\ \cdots\ g_{Rk}]$, the received signal at kth destination can be written as:

$$\begin{aligned}
\mathbf{y}_k &= \mathbf{g}_k^T\, \mathbf{t} + \mathbf{n}_k \\
&= \mathbf{g}_k^T\, \mathbf{W}^H \sum_{p=1}^{d} \mathbf{f}_p\, \mathbf{s}_p + \mathbf{g}_k^T\, \mathbf{W}^H\, \mathbf{V} + \mathbf{n}_k \\
&= \underbrace{\mathbf{g}_k^T\, \mathbf{W}^H\, \mathbf{f}_k\, \mathbf{s}_k}_{\text{desired signal componet}} + \underbrace{\mathbf{g}_k^T\, \mathbf{W}^H \sum_{p=1, p\neq k}^{d} \mathbf{f}_p\, \mathbf{s}_p}_{\text{interference component}} + \underbrace{\mathbf{g}_k^T\, \mathbf{W}^H\, \mathbf{V} + \mathbf{n}_k}_{\text{noise component}}
\end{aligned} \tag{4}$$

where $\mathbf{n}_k$ is the zero-mean noise at the kth destination with a variance of σ_n^2. Also, it is assumed that the channel coefficients between sources and relays, the channel coefficients between relays and destinations, the source signals, the relay noise and the destination noises are jointly independent.

2.1 Sum Rate Maximization Considering Consumed Power Constraint

In general, there are two ways to control relays' power. Let $P_{x,r}^{max}$ be the maximum transmit power at rth relay and P_x^{max} be the maximum sum transmit power at all the relays. Considering power control, the individual relay power constraints are [11]:

$$P_{x,r} \leq P_{x,r}^{max}, \forall r \in R \tag{5}$$

where the sum of relays' powers constraint is:

$$\sum_{r=1}^{R} P_{x,r} \leq P_x^{max} \tag{6}$$

Our goal is to achieve the weight of optimum beamforming $\{w_r^*\}_{r=1}^R$, in such a way that sum rate became maximized while the sum of consumed powers in all relay should not be greater than the certain predefined threshold. Thus, the optimization problem can be written as:

$$\begin{aligned}
&\max && \text{Sum Rate} \\
&\text{subject to} && \sum_{r=1}^{R} P_{x,r} \leq P_x^{max}
\end{aligned} \tag{7}$$

If $\sum_{r=1}^{R} P_{x,r} = P_T$, from (3) sum of consumed power in relays can be given as:

$$\begin{aligned}
P_T &= E\{\mathbf{t}^H \mathbf{t}\} \\
&= E\{\mathbf{X}^H \mathbf{W} \mathbf{W}^H \mathbf{X}\} \\
&= tr\{\mathbf{W}^H E\{\mathbf{X} \mathbf{X}^H\} \mathbf{W}\}
\end{aligned} \tag{8}$$

Now, the total consumed power can be rewritten of the form (9):

$$P_T = tr\{\mathbf{W}^H \mathbf{R}_x \mathbf{W}\} = \sum_{r=1}^{R} |w_r|^2 [\mathbf{R}_x]_{r,r}\} = \mathbf{w}^H \mathbf{D} \mathbf{w} \tag{9}$$

where $\mathbf{R}_x = E\{\mathbf{x} \mathbf{x}^H\}$ is the correlation matrix of the received signal at the relay, $\mathbf{w} = diag(\mathbf{W})$ and $\mathbf{D} = diag([\mathbf{R}_x]_{1,1}, [\mathbf{R}_x]_{2,2}, ..., [\mathbf{R}_x]_{R,R})$.

Using (2) and assumptions 1-4, Eq. (10) is given:

$$\mathbf{R}_x = \sum_{p,q=1}^{d} E\{\mathbf{f}_p \mathbf{f}_q^H\} \, E\{s_p s_q^*\} + \sigma_v^2 \mathbf{I}$$

$$= \sum_{p=1}^{d} P_p \, E\{\mathbf{f}_p \mathbf{f}_p^H\} + \sigma_v^2 \mathbf{I} \qquad (10)$$

$$= \sum_{p=1}^{d} P_p \, \mathbf{R}_f^p + \sigma_v^2 \mathbf{I}$$

where

$$\mathbf{R}_f^p = E\{\mathbf{f}_p \mathbf{f}_p^H\} \qquad (11)$$

Note that the total relay transmit power depends not only on the variances of the source-relay channel coefficients but also on the relay noise powers [10]. According to the variables that we described in the previous equations, for the system shown in Fig. 1, sum rate can be achieved from (12) [14]:

$$R_{sum} = \frac{1}{2} \sum_{k=1}^{d} \log_2 \; \det(1 + \mathbf{F}_k^{-1} \, \mathbf{T}_{kk} \, \mathbf{T}_{kk}^H) \quad (12)$$

where $\mathbf{T}_{kq} = \sum_{r\in R} g_{kr} w_r f_{rq}$, $\overline{g}_{kr} = g_{kr} w_r$ and
$\mathbf{F}_k = \sum_{q\neq k, q\in d} \mathbf{T}_{kq} \mathbf{T}_{kq}^H + \sum_{r\in R} \overline{g}_{kr} \overline{g}_{kr}^H + \sigma_n^2$.

Due to transferring data in two periods of time, 1/2 coefficient is appeared in (12). According to theorem 1 expressed in [14], then:

$$\log_2(1 + \text{TSTLR}) \leq 2R_{sum} \qquad (13)$$

Providing $\mathbf{B}_k^H \mathbf{B}_k = \mathbf{I}_{dk}$, where $\mathbf{B}_k$ is the destination beamforming.

In our investigation, destinations have single-antenna, so $\mathbf{B}_k = 1$ and the condition of theorem 1 in [14] is satisfied.

TSTLR is the sum power of desired signals to total leakage power ratio, i.e.,

$$\text{TSTLR} = \frac{\sum_{k=1}^{d} P_s^k}{\sum_{k=1}^{d} (P_n^k + P_i^k)} \qquad (14)$$

The desired signal power, interference power and noise power in terms of $\{w_r^*\}_{r=1}^{R}$ can be achieved. Using (4), for the noise power at kth receiver, we have [10]:

$$P_n^k = E\{\mathbf{v}^H \mathbf{W} \mathbf{g}_k^* \mathbf{g}_k^T \mathbf{W}^H \mathbf{v}\} + \sigma_n^2$$

$$= tr\{\mathbf{W}^H E\{\mathbf{v}\mathbf{v}^H\} \mathbf{W} E\{\mathbf{g}_k^* \mathbf{g}_k^T\}\} + \sigma_n^2 \quad (15)$$

$$= \sigma_v^2 tr\{\mathbf{W}^H \mathbf{R}_g^k \mathbf{W}\} + \sigma_n^2$$

where $\mathbf{R}_g^k = E\{\mathbf{g}_k \mathbf{g}_k^H\}$.

The kth receiver noise power can be rewritten as:

$$P_n^k = \sigma_v^2 \sum_{r=1}^{R} |w_r|^2 [R_g^k]_{rr} + \sigma_n^2$$

$$= \mathbf{w}^H \mathbf{D}_k \mathbf{w} + \sigma_n^2 \qquad (16)$$

The kth desired signal power is given by:

$$P_s^k = E\{\mathbf{g}_k^T \mathbf{W}^H \mathbf{f}_k \mathbf{f}_k^H \mathbf{W} \mathbf{g}_k^*\} E\{|s_k|^2\}$$

$$= P_k E\{\mathbf{w}^H \text{diag}(\mathbf{g}_k) \mathbf{f}_k \mathbf{f}_k^H \text{diag}(\mathbf{g}_k^*) \mathbf{w}\}$$

$$= P_k E\{\mathbf{w}^H (\mathbf{g}_k \circ \mathbf{f}_k)(\mathbf{f}_k^H \circ \mathbf{g}_k^H) \mathbf{w}\} \qquad (17)$$

$$= P_k \mathbf{w}^H E\{\mathbf{h}_k \mathbf{h}_k^H\} \mathbf{w}$$

$$= \mathbf{w}^H \mathbf{R}_h^k \mathbf{w}$$

where

$$\mathbf{h}_k = (\mathbf{g}_k \circ \mathbf{f}_k) = [g_{1k} f_{1k}, g_{2k} f_{2k}, ..., g_{Rk} f_{Rk}]^T$$

$$\mathbf{R}_h^k = P_k E\{\mathbf{h}_k \mathbf{h}_k^H\} \qquad (18)$$

It is obvious that $\mathbf{h}_k$ contains the total path gains from the kth source to its corresponding destination via R relays. Also, with using (4) and denoting $A_k = \{1,2,...,d\} - \{k\}$, the interference power is given by [10]:

$$P_i^k = E\left\{\mathbf{g}_k^T \mathbf{W}^H \left(\sum_{p,q\in A_k} \mathbf{f}_p \mathbf{f}_q^H\right) \mathbf{W} \mathbf{g}_k^*\right\}$$

$$= E\left\{\mathbf{w}^H \text{diag}(\mathbf{g}_k) \left(\sum_{p\in A_k} P_p \mathbf{f}_p \mathbf{f}_p^H\right) \text{diag}(\mathbf{g}_k^*) \mathbf{w}\right\}$$

$$= E\left\{\mathbf{w}^H \left(\sum_{p\in A_k} P_p (\mathbf{g}_k \circ \mathbf{f}_k)(\mathbf{f}_k^H \circ \mathbf{g}_k^H)\right) \mathbf{w}\right\} \quad (19)$$

$$= \mathbf{w}^H Q_k \mathbf{w}$$

where

$$\mathbf{h}_k^P = \mathbf{g}_k \circ \mathbf{f}_p$$

$$\mathbf{Q}_k = E\left\{\sum_{p\in A_k} P_p \mathbf{h}_k^p (\mathbf{h}_k^p)^H\right\} \qquad (20)$$

By using (16), (17) and (19), we can rewrite (14) as (21):

$$\text{TSTLR} = \frac{\mathbf{w}^H \left(\sum_{k=1}^{d} \mathbf{R}_h^k \right) \mathbf{w}}{\mathbf{w}^H \left(\sum_{k=1}^{d} (\mathbf{Q}_k + \mathbf{D}_k) \right) \mathbf{w} + d\sigma_n^2} \quad (21)$$

Now, the optimization problem can be rewritten as:

$$\begin{aligned} \max \quad & \text{TSTLR} \\ \text{subject to} \quad & P_T \le P_x^{\max} \end{aligned} \quad (22)$$

Using the previous equations, we can write:

$$\begin{aligned} \max \quad & \frac{\mathbf{w}^H \left(\sum_{k=1}^{d} \mathbf{R}_h^k \right) \mathbf{w}}{\mathbf{w}^H \left(\sum_{k=1}^{d} (\mathbf{Q}_k + \mathbf{D}_k) \right) \mathbf{w} + d\sigma_n^2} \\ \text{subject to} \quad & P_T \le P_x^{\max} \end{aligned} \quad (23)$$

(23) is not a convex optimization problem and may not have a low-complexity solution. Hence, it makes a difficult problem. To fix this problem, there are three ways, minimizing the denominator, maximizing the numerator, and maximizing the difference between the numerator and the denominator.

We exploit a semi-definite relaxation approach to solve a relaxed version of (23). To do so, let us define $\mathbf{X} = \mathbf{w}\mathbf{w}^H$ [6,10]. Then, the optimization problem in (23) can be rewritten as (24):

$$\begin{aligned} \max \quad & tr(\mathbf{ZX}) \\ \text{subject to} \quad & tr(\mathbf{DX}) \le P_x^{\max} \\ & \text{rank}(\mathbf{X}) = 1, \quad \mathbf{X} \ge 0 \end{aligned} \quad (24)$$

where $\mathbf{X} \ge 0$ means that $\mathbf{X}$ is a positive semi definite (PSD) matrix, and rank(.) denotes the rank of a matrix.

Maximizing TSTLR will be achieved in one of the following forms:

1- Minimizing the denominator, $\mathbf{Z} = \sum_{k=1}^{d} (\mathbf{Q}_k + \mathbf{D}_k)$

2- Maximizing the numerator, $\mathbf{Z} = \sum_{k=1}^{d} \mathbf{R}_h^k$

3- Maximizing the difference between the numerator and the denominator, $\mathbf{Z} = \sum_{k=1}^{d} \{\mathbf{R}_h^k - (\mathbf{Q}_k + \mathbf{D}_k)\}$.

The first approach is introduced in [11], namely TL. In simulations, TL method is called algorithm 1 and the two other proposed methods are named algorithm 2 and 3, respectively. As the next problem, the rank constraint in (24) is not convex. Using semi-definite relaxation technique, we remove this non-convex constraint and solve the following optimization problem:

$$\begin{aligned} \max \quad & tr(\mathbf{ZX}) \\ \text{subject to} \quad & tr(\mathbf{DX}) \le P_x^{\max} \\ & \mathbf{X} \ge 0 \end{aligned} \quad (25)$$

The optimization problem (25) is indeed convex because all constraints are in the form of linear and SDR. So, it can be efficiently solved using interior point based software tools such as the convex optimization (cvx) MATLAB toolbox [15] which produces a feasibility certificate if the problem is feasible. cvx can solve standard problems such as linear programs (LPs), quadratic programs (QPs), second-order cone programs (SOCPs), and semidefinite programs (SDPs); but compared to directly using a solver for one or these types of problems, cvx can greatly simplify the task of specifying the problem.

In SDP mode, cvx applies a matrix interpretation to the inequality operator, so that linear matrix inequalities (LMIs) and SDPs may be expressed in a more natural form.

By solving the optimization problem (25), the matrix $\mathbf{X}_{opt}$, is not necessarily of rank one. It means that the minimum value of the relaxed problem (25) only provides a lower bound on the minimum value of the original problem (24). Proof of this is available in [10]. As it is shown in [16], we can always find a rank-one solution to the relaxed problem (25) as long as $d \le 3$. Otherwise, one might resort randomization techniques to obtain a suboptimal rank-one solution. In these techniques, the optimal matrix $\mathbf{X}_{opt}$ is used to generate several suboptimal weight vectors, from which the best solution will be selected [17-19].

3. RESULTS AND DISCUSSION

In this section, the impact of various factors on the performance of the algorithms is shown. In all simulations, it is assumed that maximum sum power consumption at relays is equal to unit

($P_x^{\max} = 1$), the channel coefficients *f* and *g* are generated as identically independent distribution (i.i.d) complex Gaussian random variables with variances σ_f^2 and σ_g^2, respectively. It is also assumed that all transmitters have equal unit powers. All simulation results are averaged over 500 independent channel realizations. All MATLAB simulation codes are run on a PC with RAM=4GB, Processor: Intel (R) Core ™ i5-2400 CPU @ 3.10GHz, System Type: 64bit.

3.1 Comparison of Sum Rate Maximization Methods

In the first experiment, three methods to maximize TSTLR criterion are compared in the case of the two source-destination pairs (*d*=2), 20 relays (*R*=20) and $\sigma_f^2 = \sigma_g^2 = 10\,\mathrm{dB}$. It can be seen in Fig. 2 that the third method offers the highest sum rate with respect to two others.

Also, the required simulation time to run the above mentioned algorithms are obtained. It is assumed that *R*=20, *d*=2, $\sigma_f^2 = \sigma_g^2 = 10\,\mathrm{dB}$ and $\sigma_n^2 = -20\,\mathrm{dB}$. Algorithm 3 is considered as the reference one. The required times for algorithms 1 and 2 with respect to the third one are 1.112 and 1.003, respectively. In other words, the complexity of the proposed algorithm (algorithm 3) is lower than the two other methods (algorithm 2 and TL). Therefore, in subsequent experiments the third method is examined.

3.2 The Effect of the Number of Relays in the Performance of the Proposed Algorithm

Fig. 3 illustrates sum rate in terms of noise power σ_n^2 for different number of relays, when d=2 and $\sigma_f^2 = \sigma_g^2 = 10\,\mathrm{dB}$. It can be seen that increasing the number of relays is the reason for increasing the achievable sum rate. It is obvious that higher number of relays offers additional higher diversity. It should be noted that increasing the number of relays also introduces more computational complexity. Another important point is that although we achieve higher sum rate for a specific noise variance using higher number of relays, the sum rate difference for the same difference number of relays are not the same. For example, in $\sigma_n^2 = -5\,\mathrm{dB}$, the difference between sum rate for R=10 and R= 20 is 0.775

(b/s/Hz), for R=20 and R=30 is 0.503 (b/s/Hz) and for R=30 and R=40 is 0.404 (b/s/Hz). It is due to this fact that the main goal of beamforming is creating vectors with same phases (coherent vectors) at receiver. Since the relays' powers are limited, increasing the number of vectors will increase the sum rate. Moreover, by comparing cases containing 10 and 20 relays with each other, we find that although the number of vectors increases, the length of each vector decreases. While the number of relays increases, we tend close to a saturation case because relay's power is fixed and the total length of the vectors cannot be more than a certain amount.

Therefore, it is important to make a tradeoff between the performance and complexity based on the system requirements and the available resources.

3.3 The Effect of Channel Variance

In two following experiments, the effect of the quality of uplink and downlink channels is examined. According to [1] by increasing channel variance or equivalently the quality of channel, average received signal power is increased. In other hand, a larger variance of channel coefficients indicates a better channel or equally amplification. A careful inspection of Figs. 4 and 5 reveals that the effect of channel variance of either hop is not homogeneous in general, but the results clearly demonstrate that the proposed algorithm performs better as the channel quality improves.

3.3.1 Different uplink qualities

In Fig. 4, average achievable sum rate is plotted versus σ_n^2 where *R*=20, *d*=2 and $\sigma_g^2 = 10\,\mathrm{dB}$ for different values of σ_f^2. As can be seen in this Figure, by increasing σ_f^2 sum rate will be increased.

3.3.2 Different downlink qualities

In the case of *R*=20, *d*=2 and $\sigma_f^2 = 10\,\mathrm{dB}$ for different values of σ_g^2, the average achievable sum rate versus σ_n^2 is plotted in Fig. 5. It can be seen that increasing the quality of downlink causes increasing the achievable sum rate.
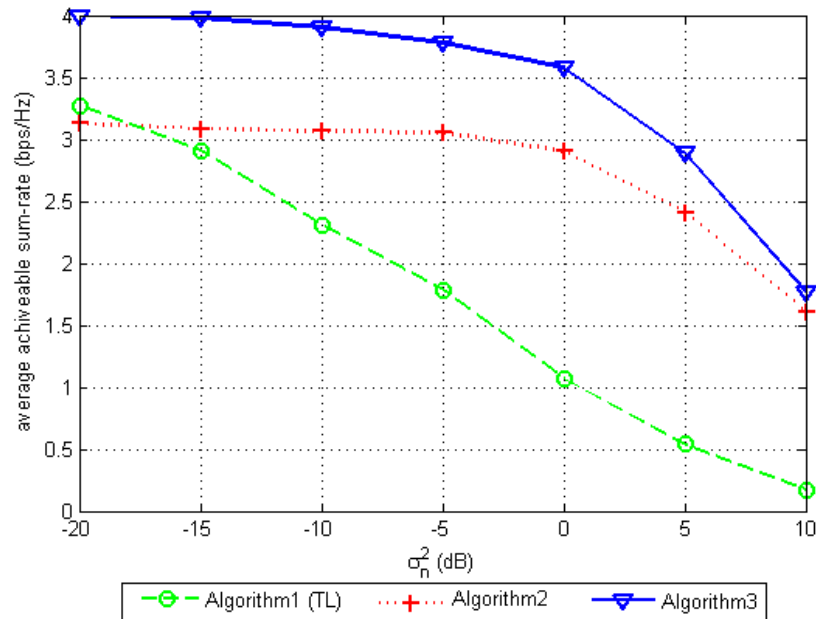
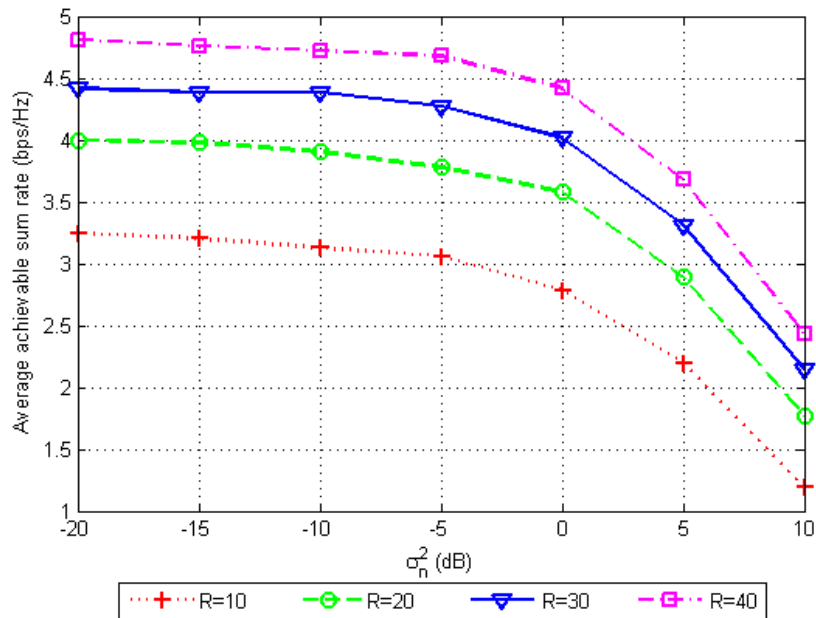**Fig. 2. Comparison of different sum rate maximization methods**

Fig. 3. Average sum rate versus noise variance for different number of relays

3.4 The Effect of Transmitter–receiver Pairs

In this experiment the effect of the number of source-destination pairs on the performance of the algorithm is investigated. In other words, we study the effect of channel interferences on the proposed algorithm. By increasing the number of source-destination pairs the interfering signal received at each destination node is also increased. The performance of the algorithm for different d is illustrated in Fig. 6 for R=20,

$\sigma_f^2 = \sigma_g^2 = 10\,\mathrm{dB}$. Fig. 6 shows that the higher the number of pairs offers the higher sum rate for certain σ_n^2. As can be seen from this Figure, for lower σ_n^2, the effect of the number of pairs is more evident and when the channel became noise dominant the effect of the transmitter – receiver pairs in increasing the sum rate will be decreased. In other words, MIMO channel tends to SISO channel.
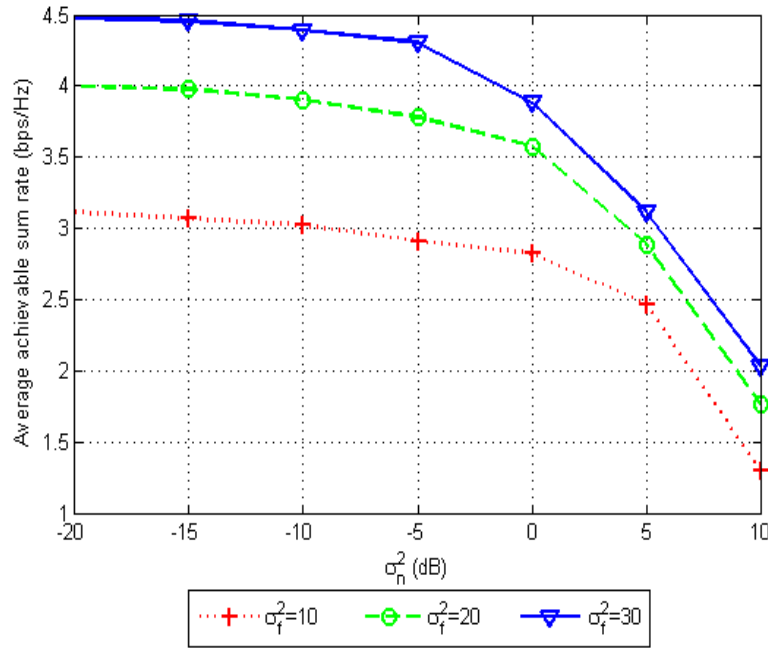

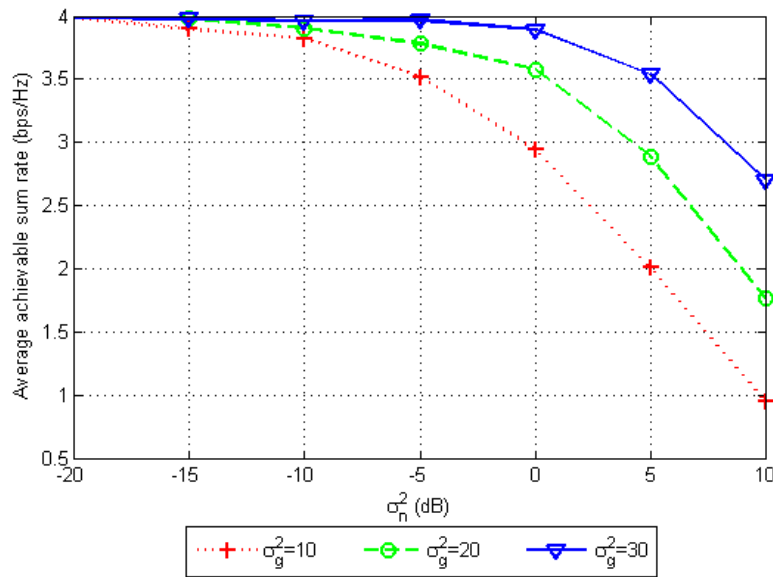**Fig. 4. Performance of the proposed algorithm for different values of** σ_f^2

Fig. 5. Performance of the proposed algorithm for different values of σ_g^2

3.5 The Effect of Imperfect CSI

In this experiment, the effect of imperfect CSI in our proposed algorithm is investigated. We assume a network with 20 relays (R=20). Also, the channels coefficients f_{rp} and $g_{r'q}$ are assumed to be independent from each other for any p, q, r and r'. Also, we assume that the channel coefficient f_{rp} can be written as $f_{rp} = \bar{f}_{rp} + ef_{rp}$ where $\bar{f}_{rp}$ is the known mean value of f_{rp} and ef_{rp} is a zero-mean random variable with variance σ_{ef}^2 [10]. We assume that ef_{rp} and $ef_{r'p}$ are independent for any $r \neq r'$. We generate $\bar{f}_{rp} = \sqrt{1-\sigma_{ef}^2}\, e^{j\theta}$, where θ is a uniform random variable which chosen from the interval $[0,2\pi]$. Since $E\left\{|f_{rp}|^2\right\}=1$, if σ_{ef}^2 is increased, the mean value, $\bar{f}_{rp}$ is decreased. This, in turn, means that the level of the uncertainty in the channel coefficient f_{rp} is increased. Similarly, we model the channel coefficient g_{rq} as $g_{rq} = \bar{g}_{rq} + eg_{rq}$ where $\bar{g}_{rq}$ is the known mean value of g_{rq} and eg_{rq} is a zero-mean random variable with variance σ_{eg}^2. We assume that eg_{rq}

and $eg_{r'q}$ are independent for any $r \neq r'$. We choose $\bar{g}_{rq} = \sqrt{1-\sigma_{eg}^2}\, e^{j\alpha}$, where α is a uniform random variable which chosen from the interval $[0,2\pi]$. Here, σ_{eg}^2 is a parameter which determines the level of uncertainty in the channel coefficient, g_{rq}. Based on this channel modeling, we can write the (r,r') entry matrices as:

$$\mathbf{R}_f^p = \bar{\mathbf{f}}\,\bar{\mathbf{f}}^{\mathrm{H}} + \mathbf{ef}\ \mathbf{I} \tag{26}$$

$$\mathbf{R}_g^p = \bar{\mathbf{g}}\,\bar{\mathbf{g}}^{\mathrm{H}} + \mathbf{eg}\mathbf{I} \tag{27}$$

$$\mathbf{R}_h^k(r,r') = P_k \mathbf{R}_f(r,r').\mathbf{R}_g(r,r') \tag{28}$$

$$\mathbf{Q}_k(r,r') = \sum_{1,k \neq p}^{d} P_p \mathbf{R}_f^p(r,r').\mathbf{R}_g^k(r,r') \tag{29}$$

In this experiment, we choose the source power equal to 0dB. The average sum rate versus σ_n^2 is plotted in the case of R=20 and d=2 for different values of σ_{ef}^2 and σ_{eg}^2 in Fig. 7. As shown in this figure, increasing the uncertainty of the channels, σ_{ef}^2 and σ_{eg}^2, is the reason for decreasing the achievable sum rate.
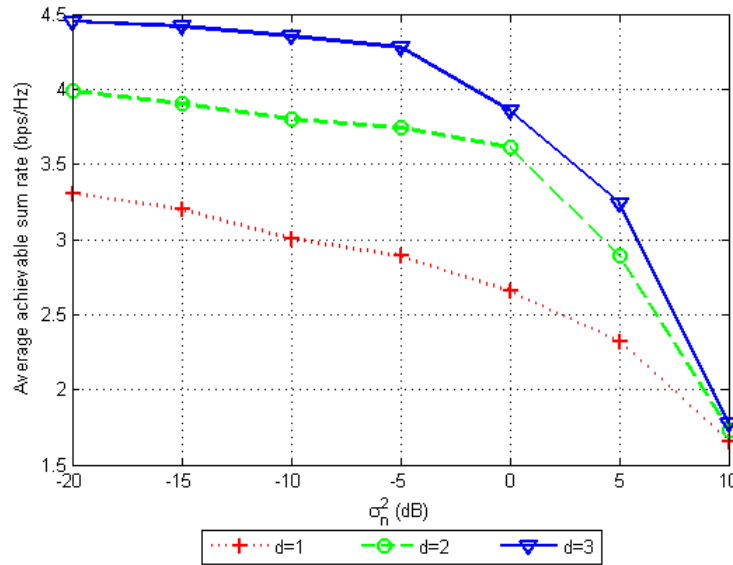

Fig. 6. Performance of the proposed algorithm for different number of source-destination pairs
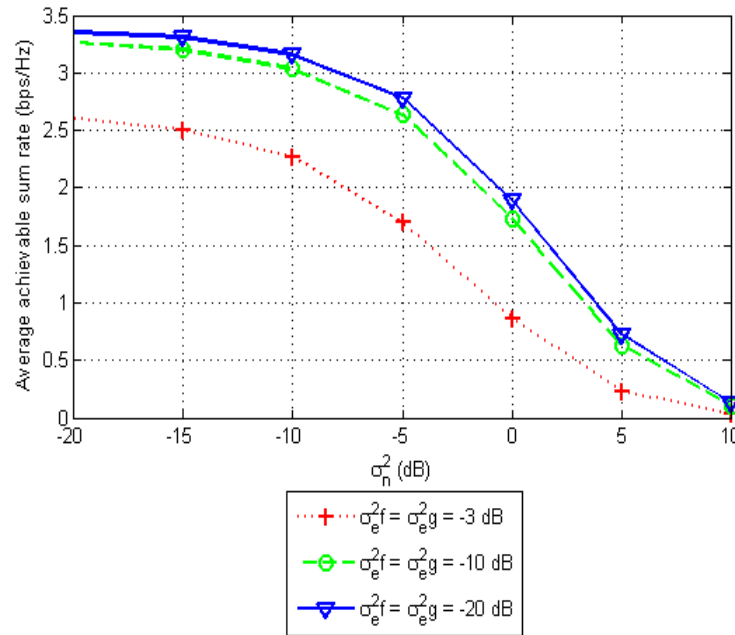
Fig. 7. Performance of the proposed algorithm for imperfect CSI

4. CONCLUSION

A network which consists of d transmitter-receiver pairs and R relay nodes is considered. The problem of multiple peer-to-peer communication is studied by considering AF scheme. In our approach, first, the sources transmit their information symbols to the relay network. Then, each relay transmits an amplitude- and phase-adjusted version of its received signal. The amplitude and phase adjustments are performed by multiplication of the relays received signal by a complex weight. The optimal weight vector plays the roles of receive and transmit beamformer at the same time. This vector is obtained through the maximizing of sum rate under constraints of total transmit relays' power. Semidefinite relaxation was used to convert this optimization problem into a semidefinite programming problem and solved the resulting convex problem efficiently using interior point methods.

Two new algorithms have been proposed to maximize total signal to total leakage ratio (TSTLR). Simulation results show that low-complex algorithm maximizes sum rate while the total consumed power in all relay nodes is not greater the certain predefined threshold. Also, when the number of relays increases the achievable sum rate increases. It was also shown that when the quality of uplink and downlink channels improves, higher sum rate can be achieved. Moreover, by increasing the number of source – destination pairs the sum rate increases, too. At last, it was shown that when uncertainty of channels increases the achievable sum rate decreases.

As we mentioned before, for $d \succ 3$, the matrix $\mathbf{X}_{opt}$ is not necessarily of rank one. So, one might resort randomization techniques to obtain a suboptimal rank-one solution. In these techniques, the optimal matrix $\mathbf{X}_{opt}$ is used to generate several suboptimal weight vectors, from which the best solution will be selected.

In this paper we assume that the second order statistics of the channel coefficients (rather than their instantaneous values) are available. Robust designs should be considered in the case of imperfect CSI for real communication systems because do not exist unlimited feedback and/or accurate channel estimation.

5. FUTURE WORK

In this paper, we assumed that all nodes have a single antenna. As the next work, we can consider the case that the nodes have multiple antennas. Another suggestion is to add individual relay power constraints and finding its effect on

the performance of the algorithm and optimization problem.

COMPETING INTERESTS

Authors declare that there are no competing interests.

REFERENCES

1. Goldsmith A. Wireless Communications. Stanford University; 2004.
2. Tse D, Viswanath P. Fundamentals of wireless communication. Cambridge University; 2005.
3. Jing Y, Jafarkhani H. Network beamforming using relays with perfect channel information. IEEE ICASSP. 2007;473–476.
4. Maadi S, Hamdi N. Power allocation and beamforming for cooperative networks. International Journal on Communication, Network and System. 2011;447-451.
5. Khandaker MRA, Rong Y. Joint power control and beamforming for interference MIMO relay channel. IEEE Wireless Communications Letters. 2011;182-187.
6. Khandaker MRA, Rong Y. Interference MIMO relay channel: Joint power control and transceiver-relay beamforming. IEEE Transactions on Signal Processing. 2012;6509-6517.
7. Rezvani SM, Safavi SH, Sadeghzadeh RA, Haghbin A, Davarpanah R. Relay power minimization approach based on general-rank beamforming for multi-antenna relaying schemes. IEEE 5th International Conference on New Technologies, Mobility and Security; 2012.
8. Hammerstrom I, Kuhn M, Wittneben A. Impact of relay gain allocation on the performance of cooperative diversity networks. IEEE 60th Vehicular Technology Conference. 2004;1815–1819.
9. Zhao Y, Adve R. Improving amplify-and-forward relay networks: Optimal power allocation versus selection. IEEE International Symposium on Information Theory. 2006;1234–1238.
10. Fazeli-Dehkordy S, Shahbazpanahi S, Gazor S. Multiple peer-to-peer communications using a network of relays. IEEE Transactions on Signal Processing. 2009;3053–3062.
11. Truong KT, Sartori P, Heath RW. Cooperative algorithms for MIMO amplify-and-forward relay networks. IEEE Transactions on Signal Processing. 2013;1272-1287.
12. Toding A, Khandaker MRA, Rong Y. Joint source and relay optimization for parallel MIMO relay networks. EURASIP Journal on Advances in Signal Processing; 2012.
13. Liu J, Shroff NB, Sherali HD. Optimal power allocation in multi-relay MIMO cooperative networks: Theory and algorithms. IEEE Journal on Selected Areas in Communications. 2012;331-340.
14. Sun C, Jorswieck E. Low Complexity sum rate maximization for single and multiple stream MIMO AF relay networks. IEEE Transactions on Information Theory; 2012: (In press).
15. Grant M, Boyd S. CVX: MATLAB software for disciplined convex programming; 2011. Available: http:// cvxr.com/cvx.
16. Huang Y, Zhang S. Complex matrix decomposition and quadratic programming. Mathematics of Operations Research; 2007.
17. Ma WK, Davidson TN, Wong KM, Luo ZQ, Ching P. Quasi maximum-likelihood multiuser detection using semi-definite relaxation with application to synchronous CDMA. IEEE Transactions on Signal Processing. 2002;912–922.
18. Sidiropoulos ND, Davidson TN, Luo ZQ. Transmit beamforming for physical-layer multicasting. IEEE Transactions on Signal Processing. 2006;2239–2251.
19. Zhang S. Quadratic maximization and semidefinite relaxation. Math. Program. 2000;453–465.