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Dynamic Model of Markets of Successive Product Generations

Joachim Kaldasch[1*]

[1]EBC Hochschule Berlin, Alexanderplatz 110178 Berlin, Germany.

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

A dynamic microeconomic model is presented that establishes the price and unit sales evolution of heterogeneous goods consisting of successive homogenous product generations. It suggests that for a fast growing supply the mean price of the generations are governed by a logistic decline towards a floor price. It is shown that generations of a heterogeneous good are in mutual competition. Their market shares are therefore governed by a Fisher-Pry law while the total unit sales are governed by the lifecycle dynamics of the good. As a result the absolute unit sales of a generation exhibit a characteristic sales peak consisting of a rapid increase followed by a long tail. The presented approach shows that the evolution of successive product generations can be understood as an evolutionary adaptation process. The applicability of the model is confirmed by a comparison with empirical investigations on successive DRAM generations.

Keywords: Product diffusion; evolutionary economics; multiple generations; competition; price evolution; DRAM market.

**Corresponding author: E-mail: joachim.kaldasch@international-business-school.de;*

1. INTRODUCTION

Presented is a microeconomic model for the price and unit sales evolution of heterogeneous goods consisting of successive homogenous product generations. Previous research on multiple generations follows two main directions of thought. One is related to technological substitution models. They can be traced back to Fisher and Pry [1] who suggested that technologies suffer from a logistic replacement dynamics of their market shares. This approach was successfully applied in particular by authors related to the Laxenburg School [2-8]. The other way of thought is to treat the evolution of successive generations as a diffusion process caused by the decision behaviour of potential buyers, introduced by Norton and Bass [9]. Mahajan and Muller [10] extended this approach by allowing potential buyers to jump between generations. Applying a discrete choice model Jun and Park [11] took the decision behaviour of potential adopters with respect to the price into account. Kim et al. [12] extended the diffusion model by including specific product characteristics. In order to enhance the forecast precision modern diffusion models of multiple product generations attempt to generalize these theories by relaxing restrictions as for example treating parameters as time dependent [13-17].

The presented model merges both attempts. It suggests that heterogeneous goods can be treated as a collection of homogenous goods of successive product generations. It is based on a generalization of the product lifecycle concept of durable and non-durable homogeneous goods established by the author to heterogeneous goods [18,19]. The unit sales of a heterogeneous good are determined mainly by two dynamic processes. On the one hand the total unit sales are governed by the product lifecycle dynamics characterized by first- and repurchase of the good. First purchase is related to the spreading of the good into the market and repurchase is due to replacement and multiple purchases. On the other hand the unit sales of homogeneous generations express their mutual competition which is the origin for the substitution of successive generations. It is shown that this process can be described by a Fisher-Pry law of the unit sales market shares. As established in a previous work the price dynamics of homogeneous goods can be regarded as a meeting process of demanded (required) with supplied (available) product units generating a price dispersion of the form of a Laplace distribution [20,21]. The mean price of this distribution declines for homogenous goods according to a logistic law for the case of a fast growing supply [18,19]. This price dynamics applies also to homogenous product generations.

The model is compared with empirical data of the price and sales evolution of Dynamic Random Access Memory (DRAM) chips. This commodity is supplied in large amounts by DRAM providers purchased mainly by manufacturers producing electronic devices. DRAMs are subject to a rapid succession of new generations. While studying this heterogeneous good Victor and Ausubel [22] characterized the sales dynamics of DRAM generations as having properties similar to fruit flies in the biological evolution. This statement is in agreement with the presented model. It can be shown here that successive product generations are governed by an evolutionary adaptation process.

The paper starts with considerations of the market dynamics of a heterogeneous good that consists of multiple homogeneous product generations by establishing their price and unit sales dynamics in polypoly markets. Explicitly derived in this work is the evolutionary replacement process of successive generations as a consequence of the market dynamics. After the theory is compared with empirical investigations of the DRAM market the paper ends with a discussion and some concluding remarks.

2. THE MODEL

The microeconomic model presented here is established for a heterogeneous good that comprises of $N(t)$ successive product generations. The key idea of this model is to consider the market dynamics as dominated by three main processes:

1. Demand and supply of product units determine the mean price of a homogeneous product generation. The mean price dynamics can be described by a Walrus equation.
2. An evolutionary substitution process determines the unit sales market shares of each product generation.
3. The total unit sales of heterogeneous goods are determined by first- and repurchase events. First purchase is governed by the spreading (diffusion) of the good into the market. Repurchase is

due to replacement and multiple purchases.

In the following chapters an analytic model is derived based on these processes.

2.1 The Dynamics of Polypoly Markets of Successive Generations

We start by studying the market dynamics of a single product generation indicated by index *i*. The demand side of the market can be specified by the total number of demanded (desired) units at time step *t* generated by potential buyers and denoted $\tilde{x}_i(t)$. The supply side is determined by the total number of supplied (available) units $\tilde{z}_i(t)$ offered by suppliers (retailers) in a polypoly market.

The total number of purchase events per unit time (total unit sales) of the *i-th* generation is indicated $\tilde{y}_i(t)$. The presented microeconomic approach is based in the idea that the purchase process can be viewed as the meeting of demanded with supplied product units. Therefore $\tilde{y}_i(t)$ must disappear if the number of demanded units $\tilde{x}_i(t)$ or supplied units $\tilde{z}_i(t)$ disappears. Hence the total unit sales of a generation can be written up to the first order as a product of both variables [23]:

$$\tilde{y}_i(t) \cong \eta_i \tilde{z}_i(t) \tilde{x}_i(t) \tag{1}$$

where the unknown rate $\eta_i \geq 0$ characterizes the mean frequency by which the meeting process generates successful purchase events. The evolution of the number of demanded and supplied units can be written as conservation relations of the form[1]:

$$\frac{d\tilde{x}_i(t)}{dt} = \tilde{d}_i(t) - \tilde{y}_i(t) \tag{2}$$

and

$$\frac{d\tilde{z}_i(t)}{dt} = \tilde{s}_i(t) - \tilde{y}_i(t) \tag{3}$$

Eq. (2) suggests that the total number of demanded units increases with the total demand rate $\tilde{d}_i(t)$ which represents the generation rate of demanded units by potential buyers. The number $\tilde{x}_i(t)$ decreases in time by the purchase of product units with the total unit sales rate $\tilde{y}_i(t)$. Eq. (3) implies that the total number of supplied units increases by the supply of product units with the total supply rate $\tilde{s}_i(t)$ and decreases with the total unit sales rate $\tilde{y}_i(t)$. The total unit sales of a heterogeneous good can be obtained from the sum over the number of current generations:

$$\tilde{y}(t) = \sum_{i=1}^{N(t)} y_i(t) \tag{4}$$

2.1.1 Demand and Supply of a Generation

In order to establish the market evolution of a generation the dynamic relations of supply and demand have to be determined. We want to take into account that demanded units have a finite mean lifetime Θ_i. That means demanded units not related to purchase events during their mean life time Θ_i disappear. This effect can be included in the demand rate $\tilde{d}_i(t)$ by writing:

$$\tilde{d}_i(t) = \tilde{d}_{0i}(t) - \frac{\tilde{x}_i(t)}{\Theta} \tag{5}$$

where $\tilde{d}_{0i}(t)$ is the generation rate of demanded units by potential buyers and the rate $1/\Theta_i$ describes the disappearance of unsatisfied demanded units. For later use we introduce the amount of demanded units $\tilde{x}_{0i}(t)$ generated by the demand rate $\tilde{d}_{0i}(t)$. It is given by:

$$\tilde{x}_{0i}(t) = \Theta_i \tilde{d}_{0i}(t) \tag{6}$$

The supply side is determined by the reproduction process. Suppliers sell product units in order to make profit. By reinvesting the profit and external money they increase the total output $\tilde{s}_i(t)$ in time. This growth process can be characterized by the variable $\gamma_i(t)$ termed reproduction parameter. It is defined by the relation between total supply flow and total unit sales of the *i-th* generation:

[1] *In order to establish a continuous model integer variables are scaled by a large constant figure such that they can be treated as small real numbers. We further demand that this scaling leads to* $\tilde{x}(t)$, $\tilde{z}(t) \leq 1$.

$$\gamma_i(t) = \frac{\tilde{s}_i(t)}{\tilde{y}_i(t)} - 1 \quad (7)$$

With this relation Eq. (3) can be rewritten as[2]:

$$\frac{d\tilde{z}_i(t)}{dt} = \gamma_i(t)\tilde{y}_i(t) \quad (8)$$

We want to confine here to the case that the supply of units of a generation evolves much faster than the number of demanded units in the considered time interval *Δt* such that:

$$\frac{d\tilde{x}_i(t)}{dt} << \frac{d\tilde{z}_i(t)}{dt} \quad (9)$$

In this case we can approximate[3]:

$$d\tilde{x}_i(t)/dt \cong 0 \quad (10)$$

Applying this relation in Eq. (2) leads with Eq. (5) to:

$$\tilde{y}_i(t) \cong \tilde{d}_i(t) = \tilde{d}_{0i}(t) - \frac{1}{\Theta_i}\tilde{x}_i(t) \quad (11)$$

In this approximation the unit sales of a generation are (nearly) equal to the generation rate of demanded units diminished by the rate $\tilde{x}_i(t)/\Theta_i$. Applying Eq. (1) and Eq. (6) in Eq. (11) we get for the total number of demanded units:

$$\tilde{x}_i(t) = \frac{\tilde{x}_{0i}(t)}{1+\Theta_i\eta_i\tilde{z}_i(t)} \quad (12)$$

Expanding this equation for small $\tilde{z}_i(t)$ yields:

$$\tilde{x}_i(t) \cong \tilde{x}_{0i}(t)\left(1-\Theta_i\eta_i\tilde{z}_i(t)\right) \quad (13)$$

In order to determine the time evolution of $\tilde{z}_i(t)$ we further apply Eq. (13) in Eq. (1) and obtain from Eq. (8):

$$\frac{d\tilde{z}_i(t)}{dt} = \alpha_i(t)\tilde{z}_i(t) - \Theta_i\eta_i\alpha_i(t)\tilde{z}_i(t)^2 \quad (14)$$

with:

$$\alpha_i(t) = \eta_i\tilde{x}_{0i}(t)\gamma_i(t) \quad (15)$$

In order to solve Eq. (14) the time dependent function $\alpha_i(t)$ is replaced by its time average:

$$\alpha_i = \frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t}\alpha_i(t)dt \quad (16)$$

Then Eq. (14) turns into a logistic differential equation with constant coefficients. The number of supplied units of the *i-th* generation can be given by:

$$\tilde{z}_i(t) = \frac{z_{\max i}}{1+C_{zi}e^{-\alpha_i t}} \quad (17)$$

with the integration constant C_{zi} and:

$$z_{\max i} = \frac{1}{\eta_i\Theta_i} \quad (18)$$

For the case of a fast growing supply $\gamma_i \sim \alpha_i > 0$, Eq. (17) predicts that the total number of available units $\tilde{z}_i(t)$ increases in time according to a logistic law until $\tilde{z}_i(t) = z_{maxi}$.[4] If on the other hand $\alpha_i < 0$, the number of available units decreases exponentially until they disappear.

The total unit sales Eq. (11) have with Eq. (12) and Eq. (17) the form:

$$\tilde{y}_i(t) = \frac{\tilde{d}_{0i}(t)z_i(t)}{z_{\max}} = \frac{\tilde{d}_{0i}(t)}{1+C_{zi}e^{-\alpha_i t}} \quad (19)$$

This relation suggests that the sales evolution is determined on the one hand by the generation rate of demanded units $\tilde{d}_{0i}(t)$ and on the other hand the evolution of available units $\tilde{z}_i(t)$. Since $\tilde{z}_i(t)$ is usually a small figure at introduction of a generation, the number of sold units per unit time is smaller than the generation rate of demanded units $\tilde{d}_{0i}(t)$, known as lost sales [19].

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[2] *Note that the finite lifetime of the good is neglected here (see [18]).*

[3] *This simplification is known as adiabatic approximation.*

[4] *Note that z_{maxi} is the maximum number of available units without supply constraints. If there are (external) supply constraints the maximum number of available units has a smaller magnitude $z'_{maxi} < z_{maxi}$.*

2.1.2 Price evolution of a homogeneous generation

In order to establish the price evolution of the *i-th* generation we introduce the number of demanded and supplied units $x_i(p,t)$ and $z_i(p,t)$ as accumulated functions over the price *p* [21]. Generalizing Eq. (1) we assume that the number of sold units in a given price interval must disappear if the corresponding numbers $x_i(t,p)$ or $z_i(t,p)$ disappear. Hence the price dependent unit sales are up to first order proportional to both variables:

$$y_i(t,p) \cong \eta_i x_i(t,p) z_i(t,p) \qquad (20)$$

where the meeting rate η_i is treated as price independent. The price dispersion of sold units is determined by the probability density:

$$P_i(t,p) = \frac{y_i(t,p)}{\tilde{y}_i(t)} \qquad (21)$$

As established in [21] the price dispersion of homogeneous goods can be approximated with Eq. (20) for short time horizons by a symmetric Laplace distribution. For a homogenous generation it has the form:

$$P_i(p) \cong \frac{1}{2\sigma_i} e^{-\frac{|p_i - \mu_i|}{\sigma_i}} \qquad (22)$$

where μ_i is the mean price of the generation and σ_i is related to the standard deviation of the price distribution by:

$$Std(P_i(p)) = \sqrt{2}\sigma_i \cong \sqrt{2}(\mu_i - \mu_{mi}) \qquad (23)$$

The minimum mean price of the *i-th* generation $\mu_{mi}>0$ indicates a limit beyond which the production is not profitable. Also established is that the mean price evolution can be described by a Walrus equation [21]. For a homogenous good it has the form:

$$\frac{1}{\mu_i - \mu_{mi}} \frac{d\mu_i}{dt} = H_i \left(\frac{d\tilde{x}_i}{dt} - \frac{d\tilde{z}_i}{dt} \right) \qquad (24)$$

while $H_i>0$ is treated as a constant. This relation suggests that the mean price increases if there is an excess increase of demanded units per unit time and decreases for an excess increase of supplied units [18]. It can be used to characterize the evolution of the mean price of a generation. For this purpose we take advantage from Eq. (10) and approximate:

$$\frac{1}{\mu_i(t) - \mu_{mi}} \frac{d\mu_i(t)}{dt} \cong -H_i \frac{d\tilde{z}_i(t)}{dt} \qquad (25)$$

That means, for a fast evolving supply market the mean price is essentially governed by the evolution of supplied units. Applying Eq.(14) we further get:

$$\frac{d\mu_i(t)}{dt} \cong -H_i \alpha_i \tilde{z}_i(t) (\mu_i(t) - \mu_{mi}) \qquad (26)$$

while higher order terms in $\tilde{z}_i(t)$ are neglected. The mean price evolution depends on the sign of α_i. For $\alpha_i<0$, the mean price exhibits an exponential increase proportional to $\tilde{z}_i(t)$ due to supply shortage. The mean price approaches a maximum magnitude when $\tilde{z}_i = 0$. For $\alpha_i>0$, however, $\mu_i(t)$ declines as a consequence of the excess supply. The stationary solution of this relation is given either by $\mu=\mu_m$ or $\tilde{z} = z_{\max}$. Since we confine here to polypoly markets the first case is not further considered here.

For $\mu(t)>\mu_m$, Eq. (25) can be written as:

$$\int \frac{d\mu_i(t)}{\mu_i(t) - \mu_{mi}} \cong -H_i \int d\tilde{z}_i(t) \qquad (27)$$

and we readily obtain:

$$\mu_i(t) = \mu_{0i} e^{-H_i \tilde{z}_i (t - t_{0i})} + \mu_{mi} \qquad (28)$$

where t_{0i} indicates the introduction time step of the *i-th* generation. The model suggests therefore that for the considered market constellation the mean price of a generation declines with increasing $\tilde{z}_i(t)$ according to the logistic law Eq.(17). For $\tilde{z}_i(t) \to z_{\max i}$ the mean price approaches a floor price:

$$\mu_{fi} = \mu_{0i} \exp(-H_i z_{\max i}) + \mu_{mi} \qquad (29)$$

The introduction mean price of the good $\mu(0)$ is defined by:

$$\mu_i(0) = \mu_{0i}\exp(-H_i\tilde{z}_i(0)) + \mu_{mi} \quad (30)$$

while $\tilde{z}_i(0) \neq 0$.

2.2 Market Share Evolution of Successive Generations

We want to continue by evaluating the evolution of the unit sales market share of the *i-th* generation. For this purpose we take the time derivative of Eq. (1). Applying Eq. (10) yields:

$$\frac{d\tilde{y}_i(t)}{dt} \cong \eta_i \tilde{x}_{0i}\frac{d\tilde{z}_i(t)}{dt} \quad (31)$$

where the slowly varying number of demanded units is approximated by $\tilde{x}_{0i}$. With Eq. (15) this relation can be written as:

$$\frac{d\tilde{y}_i(t)}{dt} = \alpha_i(t)\tilde{y}_i(t) \quad (32)$$

That means the unit sales evolution of a generation is governed by the growth rate $\alpha_i(t)$. The market share of the *i-th* generation is defined by:

$$m_i(t) = \frac{\tilde{y}_i(t)}{\tilde{y}(t)} \quad (33)$$

Taking the time derivative we get:

$$\frac{dm_i(t)}{dt} = \frac{1}{\tilde{y}(t)}\frac{d\tilde{y}_i(t)}{dt} - \frac{\tilde{y}_i(t)}{\tilde{y}(t)^2}\frac{d\tilde{y}(t)}{dt} \quad (34)$$

Inserting Eq. (32) in this relation yields:

$$\frac{dm_i(t)}{dt} = \left(\alpha_i(t) - \langle\alpha(t)\rangle\right)m_i(t) \quad (35)$$

where:

$$\langle\alpha(t)\rangle = \sum_i \alpha_i(t)m_i(t) \quad (36)$$

is the mean unit sales growth rate of the good. Eq. (35) is a replicator equation of the market share of a generation. It expresses the competition between different product generations while the fitness of a generation:

$$f_i(t) = \alpha_i(t) - \langle\alpha(t)\rangle \quad (37)$$

is determined following Eq. (15) by the preference for a generation, the mean reproduction parameter characterizing the financial success in the reproduction process and the mean number of demanded units.

2.2.1 The replacement process of successive generations

A consequence of the mutual competition between successive generations is the tendency to replace each other.[5] In order to describe the replacement process we take advantage from the replicator dynamics Eq. (35) and write:

$$\frac{1}{m_i}\frac{dm_i}{dt} = \frac{d\ln(m_i)}{dt} = f_i(t) \quad (38)$$

The stationary solution of Eq.(38) becomes:

$$m_i = \begin{cases} 1 & if \;\; f_i(t) = 0 \\ 0 & if \;\; f_i(t) \neq 0 \end{cases} \quad (39)$$

This relation states that for a constant fitness advantage just one generation with $\alpha_i = \langle\alpha\rangle$ and $m_i=1$ survives the competition process after sufficient time. In order to establish the market share evolution of successive product generations the predecessor generation with index *j* is diminished from Eq. (37) such that:

$$\frac{d}{dt}\ln\left(\frac{m_i}{m_j}\right) = \alpha_i(t) - \alpha_j(t) = f_{ij}(t) \quad (40)$$

where $f_{ij}(t)$ is termed the fitness advantage with respect to the *j-th* generation. The relation between the two market shares becomes:

$$\frac{m_i(t)}{m_j(t)} = \frac{m_i(t_{0i})}{m_j(t_{0i})}\exp\left(\int_{t_{0i}}^{t_{0i}+\Delta t'} f_{ij}(t')dt'\right) \quad (41)$$

where $\Delta t'$ is the time period in which the generation *i* is in competition with generation *j*. The fitness advantage can be written as the sum of a mean fitness advantage f_{ij} and time dependent fluctuations $\delta f_{ij}(t)$ as:

$$f_{ij}(t) = f_{ij} + \delta f_{ij}(t) \quad (42)$$

[5] *We consider the case that suppliers continue to provide product units of the i-th generation even when the next generation enters the market.*

where the mean fitness advantage is the averaged over the time interval *Δt'*:

$$f_{ij} = \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t'} \left(\alpha_i(t) - \alpha_j(t) \right) dt \qquad (43)$$

In order to keep the model simple we confine here to the case that the mean fitness advantage dominates over time dependent fluctuations:

$$f_{ij} >> \delta f_{ij}(t) \qquad (44)$$

In this case the integral in Eq. (41) can be reduced to the dominant contribution and becomes:

$$\frac{m_i(t)}{m_j(t)} \cong \frac{m_i(t_{0i})}{m_j(t_{0i})} e^{f_{ij}(t-t_{0i})} = e^{f_{ij}(t-t_{0i})+\kappa_{ij}} \qquad (45)$$

with:

$$\kappa_{ij} = \ln\left(\frac{m_i(t_{0i})}{m_j(t_{0i})} \right) \qquad (46)$$

Applying the identity [24]:

$$m_i(t) = \frac{m_i(t)}{\sum_j m_j(t)} = \frac{1}{1 + \sum_{j \neq i} \frac{m_j(t)}{m_i(t)}} \qquad (47)$$

we get for the time evolution of the *i-th* market share:

$$m_i(t) = \frac{1}{1 + \sum_{j \neq i} e^{-f_{ij}(t-t_{0i})+\kappa_{ij}}} \qquad (48)$$

If the competition takes place essentially between neighbouring generations this relation can be simplified to the two generation case with $m_j=1-m_i$. It turns into the well-known Fisher-Pry substitution law:

$$m_i(t) = \frac{1}{1 + e^{-f_{ij}(t-t_{0i})+\kappa_{ij}}} \qquad (49)$$

The presented model suggests therefore that the replacement process of successive generations is governed for the case of a constant fitness advantage by a Fisher-Pry law. It has its origin in the change of the preference magnitude by potential buyers η_i with the introduction of a new product generation in Eq. (15). The fitness advantage is $f_{ij}>0$ for $j=i-1$ and $f_{ij}<0$ for $j=i+1$. Hence, the market share evolution separates into two branches. One is related to the competition with the predecessor generation $j=i-1$ and one with the successor $j=i+1$.

Applying Eq. (33) the unit sales evolution of the *i-th* generation is with Eq. (49) determined by:

$$\tilde{y}_i(t) = \frac{\tilde{y}(t)}{1 + e^{-f_{ij}(t-t_{0i})+\kappa_{ij}}} \qquad (50)$$

while the total unit sales of the good are determined by Eq. (4).

2.2.2 Unit sales evolution of a heterogeneous good

In order to estimate the total unit sales $\tilde{y}(t)$ of the heterogeneous good we apply the product lifecycle concept [18,19]. This concept suggests that the total unit sales of a good can be separated into characteristic phases: the introduction, growth, maturity and decline phase. The total unit sales must be determined by first and repurchase of the good. First purchase is governed by the diffusion process described by:

$$n(t) = \frac{N_A(t)}{M} \qquad (51)$$

where $N_A(t)$ is the cumulative number of adopters and M the market potential, i.e. the number of all possible adopters of the good. The diffusion process can be modelled in a first approximation by the Bass model [25] given by:

$$\frac{dn(t)}{dt} = \mathrm{A}(n_{\max} - n(t)) + \mathrm{B}n(t)(n_{\max} - n(t)) \qquad (52)$$

where n_{max} indicates the maximum market penetration. The Bass model suggests that the spreading of the good into the market is related to two spreading waves, a fast and a slow one. The fast one is described by the first term in Eq. (53) indicating a spontaneous purchase by potential adopters and mediated by mass media. It is proportional to the so-called innovation rate A and the number of remaining potential adopters n_{max}-$n(t)$. The second term describes a much slower spreading process due to social contagion, where the number of adopters increases with an imitation rate B proportional to the product of the number of current and potential adopters. The evolution of the market penetration due to Bass diffusion becomes:

$$n(t) = \frac{1-e^{-(A+B)t}}{1+\frac{B}{A}e^{-(A+B)t}} n_{\max} \quad (53)$$

with the corresponding first purchase total unit sales:

$$\tilde{y}_f(t) = \frac{dn(t)}{dt} = \frac{A(A+B)^2 e^{-(A+B)t}}{\left(A+Be^{-(A+B)t}\right)^2} n_0 \quad (54)$$

Repurchase events of the good must be proportional to the current number of adopters $n(t)$ and a repurchase rate $\xi(t)$ characterizing the average number of units purchased per unit time and adopter. Hence:

$$\tilde{y}_r(t) = n(t)\xi(t) \quad (55)$$

The repurchase process consists of multiple and replacement purchase [26,27]. Starting from zero, after sufficient time the repurchase rate must be proportional to $1/\tau$, where τ is the mean lifetime of the good. Denoting the mean number of multiple purchased units during this lifetime by ι the repurchase rate approaches a maximum magnitude $\xi_{max}=\iota/\tau$. Describing the repurchase rate evolution as a growth process towards this limit the time evolution of $\xi(t)$ can be modelled as:

$$\frac{d\xi(t)}{dt} = a\xi(t)(\xi_{\max} - \xi(t)) \quad (56)$$

with the stationary solution $\xi=\xi_{max}$ and the repurchase growth rate a. The repurchase rate evolution becomes a logistic law of the form [18,19]:

$$\xi(t) = \frac{\xi_{\max}}{1+C_{\xi}e^{-at}} \quad (57)$$

with the free parameters a, ξ_{max} and C_{ξ}. The total unit sales of the heterogeneous good turns into:

$$\tilde{y}(t) = \tilde{y}_f(t) + \tilde{y}_r(t) = \frac{dn(t)}{dt} + n(t)\xi(t) \quad (58)$$

Applying Eq. (4) we can also write for the total unit sales of the heterogeneous good:

$$\tilde{y}(t) = \sum_{j\neq i}\tilde{y}_j(t) + \tilde{y}_i(t) = (1-m_i(t))\tilde{y}(t) + m_i(t)\tilde{y}(t) \quad (59)$$

For $m_i(t)\approx 1$, the total unit sales can be approximated by the unit sales of the *i-th* generation $\tilde{y}(t) \approx \tilde{y}_i(t)$. This is the case in the introduction period of a good where the number of generations is small and their market shares are relatively high. However, with an increasing number of generations the market share of each generation necessarily declines. For $m_i(t)<<1$, the impact of a generation on the evolution of the total unit sales described by Eq. (58) is small. We want to confine the model here to the latter situation. That means we consider the case where the impact of a single generation on the total sales is sufficiently small to be neglected.[6] Under this condition the unit sales of a generation can be specified by applying Eq. (58) in Eq. (50).

The description of the lifecycle of a heterogeneous good consisting of $N(t)$ homogeneous successive generations introduced at time step t_{0i} requires a minimum number of free parameters. The mean price of the *i-th* generation involves the specification of the number of available units $\tilde{z}_i(t)$ determined by three free parameters z_{maxi}, α_i and C_{zi}. For the mean price given by Eq. (29) three additional parameters are required μ_{0i}, μ_{fi} and H_i (For simplicity we can set $\mu_{mi}=0$). The market shares of the generations are determined by the fitness

[6] *Only if a generation is a major success the total sales are considerably disturbed by this generation. In this case the model has to be extended by a perturbation approach not considered here.*

with the neighbouring generations f_{ij} and the free parameters κ_{ij}. The total unit sales evolution of the heterogeneous good is governed by the parameters *A, B*, n_0, *a*, ξ_{max} and C_ξ.

3. COMPARISON WITH EMPIRICAL RESULTS

Before comparing the model with empirical data we want to summarize the model predictions. The theory suggests that:

1. The short term price dispersion $P_i(p)$ of a homogeneous generation has the form of the Laplace distribution Eq. (22).
2. The mean price evolution of the *i-th* generation $\mu_i(t)$ is governed by a logistic decline in the case of a fast growing supply described by Eq. (28).
3. Competitive successive product generations are governed by a Fisher-Pry law of the market shares $m_i(t)$ determined by Eq. (49).
4. The total unit sales of homogenous generations can be approximated by Eq. (50).
5. The total unit sales of a heterogeneous good can be given by the lifecycle dynamics of the good Eq. (58).

In order to illustrate the applicability of the model, it is applied to DRAMs as a heterogeneous good comprising of homogeneous technological generations. DRAMs are memory chips finding their main application in computers, primarily in PCs and servers. The permanent creation of new DRAM generations with increasing memory were generating up and down cycles of the unit sales, termed sales peaks. The DRAM market is one of the most closely watched markets of all integrated circuit categories. Several papers studied the technological evolution of DRAMs, trying to understand the dynamics of memory chip generations from different economic and technological perspectives [28-31]. We want to discuss here the empirical mean price and unit sales evolution of the DRAM market and compare them with the presented model.

3.1 The Mean Price Evolution of Successive DRAM Generations

DRAMs of a specific generation are standardized memory chips determined only by their memory size and the price. DRAM generations can therefore be treated as homogeneous. The model predicts that the short term price dispersion of homogeneous goods have the form of a Laplace distribution. This statement can, however, not be compared with the model statement since empirical data of the price dispersion of DRAM generations are unfortunately not available.

We focus here on a consideration of the mean price evolution of the first DRAM generations. As a proxy for $\mu_i(t)$ we take advantage from empirical data of the average selling price (ASP) displayed in Fig. 1 in a half-logarithmic plot [22,29]. Also displayed is a fit of Eq. (28) with the parameters summarized in Table 1. The model suggests that the mean price of a product generation decreases according to a logistic law approaching a constant floor price after sufficient time for a fast growing supply market. However, the DRAM market was subject to considerable external perturbations, namely the so-called Semiconductor Trade Arrangements (STA). These Arrangements are attempts of the US-government to reduce the dominance of Japanese chipmakers by limiting the DRAM supply [32,33]. The first STA started 1986 and ended 1991 indicated in Fig. 3. As a result the ASP of the DRAM generations deviates from the logistic price evolution with constant coefficients (lines). The price increase compared to the predicted lines can be interpreted as a consequence of the constrained supply in this period.[7]

In 1991, the STA was extended for another five years. In the second STA period the empirical price of the DRAM generations approaches floor prices caused by the external supply constraints. The constraint supply in the second STA period can be taken into account in this model by assuming that the number of available units $\tilde{z}(t)$ evolves in two logistic waves with constant parameters separated by the time step $t_1=1995$ at the end of the second STA period. For $t<t_1$, the supply growth is assumed to be governed by $\alpha_i^{(1)}$, $z'^{(1)}_{max}$ and $C_z^{(1)}$. For $t \geq t_1$, the parameters are $\alpha_i^{(2)}$, $z_{max}^{(2)}$ and $C_z^{(2)}$ while $\tilde{z}(t)$ starts at $z'^{(1)}_{max}$. With the parameters given in Table 1 the empirical price evolution in Fig. 1 can be fitted as consisting of two logistic price decline periods.

[7] *Note that the first generation (4K) exhibits a considerable increase of the price after introduction of the next generation (16K). This is due to the reduction of the total output by the manufactures. It corresponds in this model to the case $\alpha_i<0$ suggesting an exponential increase of the mean price. This effect is not further discussed here.*

Table 1. Characteristic parameters of the mean price (Eq. (28)) of the first DRAM generations displayed in Fig. 1

Parameter	i=4K	i=16K	i=64K	i=256K	i=1M	i=4M	i=16M	i=64M
t_{0i}	1974	1976	1978	1982	1985	1988	1991	1978
α [$year^{-1}$]	1	0.6	0.6	0.8				1.4
z_{max}	1	1	1	1				1
C_z	1.5	1.5	2.1	2.1				50
$z'^{(1)}_{max}$					0.8	0.8	0.6	
$C_z^{(1)}$					1.2	1	0.8	
$\alpha^{(1)}$ [$year^{-1}$]					0.6	0.8	1	
t_1					1995	1995	1995	
$z_{max}^{(2)}$					0.2	0.2	0.4	
$C_z^{(2)}$					5	40	22	
$\alpha^{(2)}$ [$year^{-1}$]					0.7	3	1.9	
μ_0 (US$)[8]	120	613	3120	2950	2160	18850	4170	440
μ_f (US$)	1.9	1.0	1.1	1.5	0.7	1.5	2.2	4.3
H	4.1	6.4	7.9	7.5	8	9.4	7.5	4.6

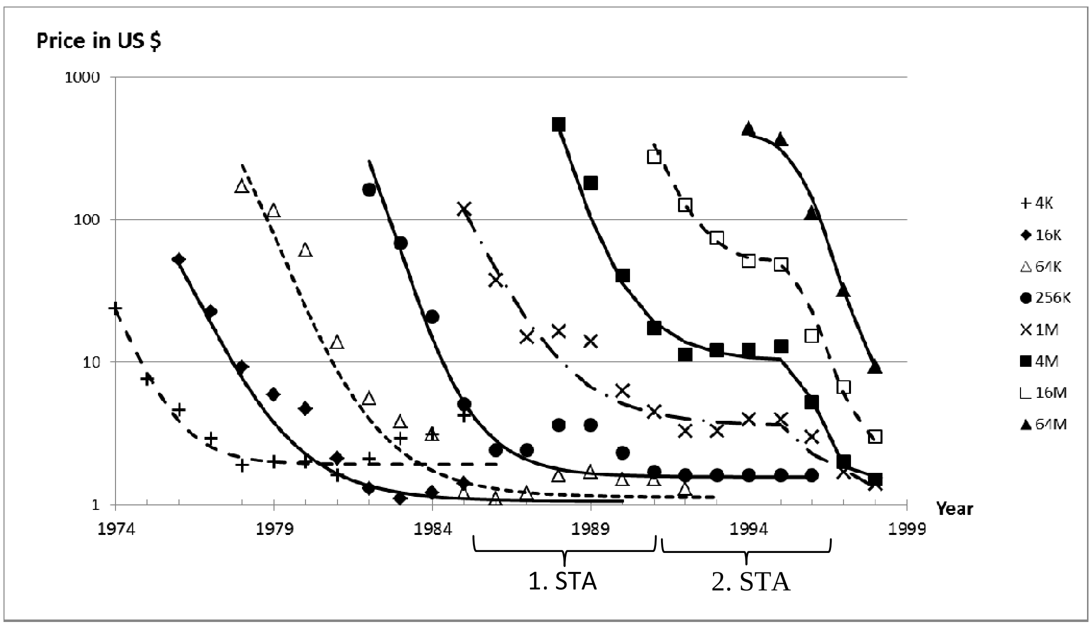

Fig. 1. Evolution of the average sales price (ASP) of the first DRAM generations [22,29]. The lines are a fit of Eq.(28) with the free parameters in Table 1. Indicated are the first and second STA periods

3.2 The Unit Sales Evolution of Successive DRAM Generations

A main result of the model is the prediction of the market share evolution of successive generations is a consequence of their mutual competition. Only the first generation is free of competition and therefore equivalent to the evolution of the total unit sales. The model suggests that in the introduction period of a good the market shares of the generations are relatively high. Displayed in Fig. 2 are the empirical market shares of the investigated DRAM generations [22,29,33]. It can be seen that the maximum market share of the generations declines in time. While the first generation starts with maximum market share $m_{4K}=1$ and therefore dominated the initial stage of the unit sales evolution of the good the

[8] *Note that a high μ_0 expresses a high magnitude of available units at t_{0i} in this model.*

maximum magnitude declines in time until $m_i \approx 0.5$ for the last investigated generations.

If competition between generations takes place Eq. (49) suggests that setting out $ln(m_i/(1-m_i))$ against time the data should arrange along straight lines, while the slope expresses the fitness advantage f_{ij} (Fisher-Pry plot). Displayed in Fig. 3 are the historical unit sales market shares of successive DRAM generations in a Fisher-Pry plot. As can be seen in this figure the unit sales evolution of successive DRAM generations has the form of linear functions. This result suggests that the market share evolution of successive DRAM generations is governed by a Fisher-Pry-law. Applying a regression analysis of the empirical data the fitness parameters f_{ij} can be obtained. They are summarized in Table 2.

Shown in Fig. 4 are the empirical absolute unit sales of the DRAM generations together with the total sales. Since the unit sales of the first generations are small Eq.(58) is applied to later time steps where repurchase dominates. The fit of the total unit sales $\tilde{y}(t)$ with Eq. (58) is displayed in Fig. 4 by the solid line. Taking advantage from the parameters of the regression fit in Table 2 the unit sales $\tilde{y}_i(t)$ of the DRAM generations can be evaluated from Eq. (50) also displayed in Fig. 3 (lines). As can be expected from the model in particular the higher generations are well described by the presented approach.

Table 2. The fitness advantage f_{ij} (fitness disadvantage with the next generation f_{ji}) of the successive DRAM generations from a linear regression fit of the data in the Fisher-Pry plot Fig. 2

Parameter	f_{ij}	f_{ji}
i=16K j=4K	*1.49*	*-1.23*
i=64K j=16K	*1.38*	*-1.22*
i=256K j=64K	*1.11*	*-0.84*
i=1M j=256K	*1.26*	*-0.65*
i=4M j=1M	*1.39*	*-0.59*
i=16M j=4M	*1.36*	*-0.74*
i=64M j=16M	*1.49*	*-0.57*
i=128M j=64M	*1.14*	*-0.63*
i=256M j=128M	*1.37*	*-0.52*
i=512M j=256M	*1.83*	*-0.60*
i=1G j=512M	*1.61*	*-0.59*
i=2G j=1G	*1.66*	*-0.59*

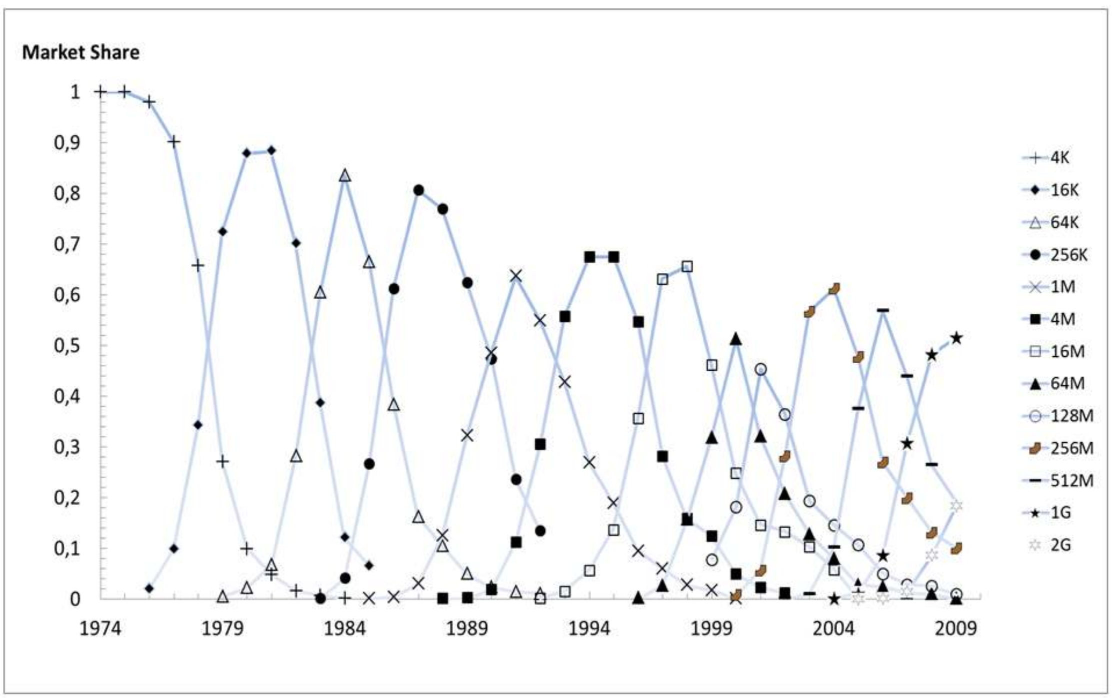

Fig. 2. Market shares of the investigated DRAM generations [22,29,33]

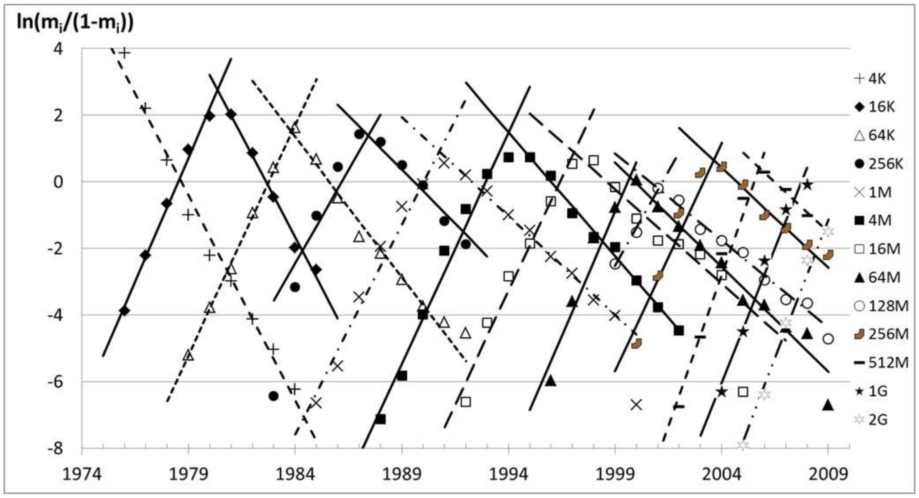

Fig. 3. Unit sales of the first DRAM generations in a Fisher-Pry plot. The lines are regression curves with the parameters summarized in Table 2

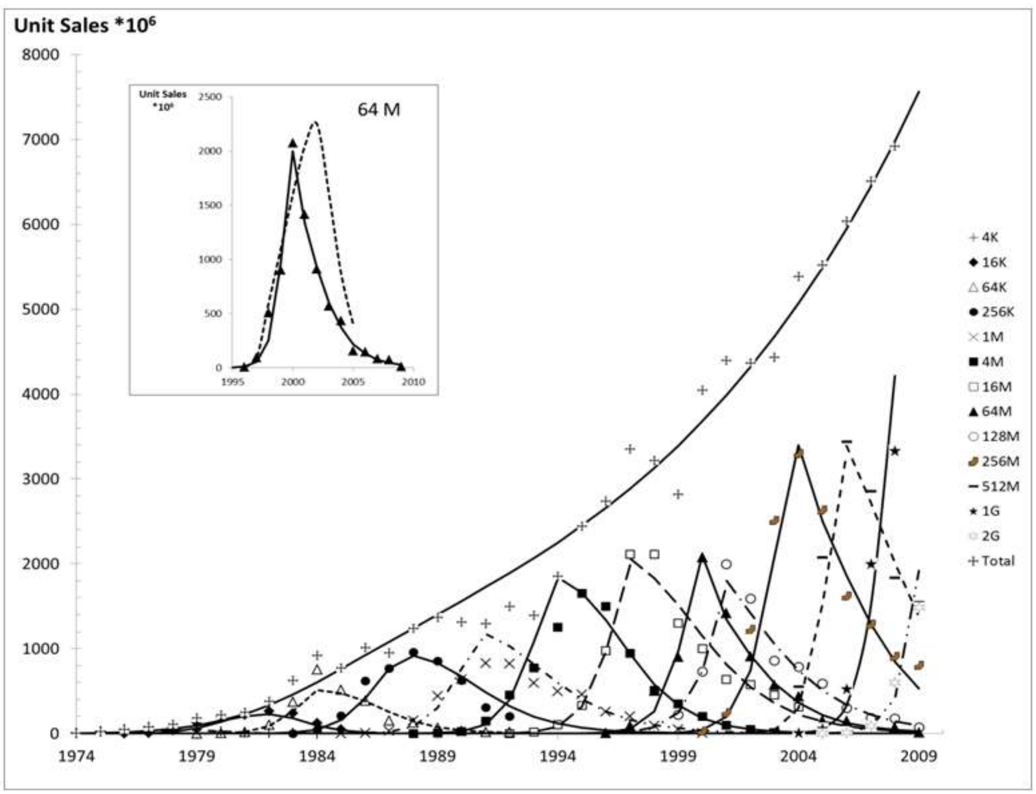

Fig. 4. Displayed are the absolute empirical unit sales of DRAM generations [22,29,33]. The total unit sales are described by Eq.(58) with the parameters *A=0.01, B=0.4, n_{max}=1, a=0.08, ξ_{max}=5*10^{10}* and *C_ξ=100*. The theoretical units sales (lines) are obtained by applying Eq.(50) with the parameters in Table 2. The insert shows a comparison of the empirical 64 MByte sales peak (triangles) with the presented model (solid line) and a prediction by ref. [22] (dashed line)

4. DISCUSSION

The key feature of heterogeneous goods is the presence of multiple generations with substantial fitness advantages f_{ij} (Eq. (44)). Only in this case generations suffer from a substitution process and the unit sales evolution of a generation may differ considerably from the total unit sales of the good. If fitness advantages mainly fluctuate in time multiple generations are not replaced and the price and unit sales dynamics of a generation are similar to that of the good. Such a good can be characterized as nearly homogeneous. Heterogeneous goods on the other hand undergo a substantial technological evolution [34]. The generations exhibit the suggested logistic replacement dynamics of the market shares due to considerable fitness advantages as can be found for example for personal computers [35], video cassette recorders (Betamax, VHS) [23], locomotives [34], vessels [34] etc. Here the theory is exemplified at a series of successive generations of DRAM memory chips.

The model predicts the unit sales of the heterogeneous good and in addition the unit sales and mean price evolution of the generations. It suggests that for the case of an unconstraint growth of the supply side the mean price of a generation declines according to a logistic law. This can be seen for the first DRAM generations in Fig. 1. However, supply constraints due to the Semiconductor Trade Arrangements perturb the empirical mean price evolution. Taking supply constraints as limitations of the number of supplied units into account the evolution of the mean price turns into successive logistic decline waves also displayed in Fig. 1.

The initial stages of the life cycle of a heterogeneous good are determined by the spreading of the good into the market. In this period the number of generations is usually small and their unit sales entails this process [36]. Previous research on successive product generations are based on the idea that the unit sales evolution of successive generations can be viewed as governed by diffusion processes [9,22,32]. However, in later stages of the lifecycle repurchase dominates the total unit sales. With an increasing number of generations the unit sales of the generations cannot be treated as independent because mutual competition governs their development.

In order to illustrate the difference between the two interpretations the empirical unit sales of 64 MByte DRAMs are compared on the one hand with a prediction of a diffusion model established by Victor and Ausubel [22] (dashed line) and on the other hand with the presented model (solid line) shown in the insert of Fig. 3. Diffusion models with constant coefficients lead generally to symmetric sales peaks. The presented model based on the Fisher-Pry law suggests, however, that a unit sales peak consists of a steep rise followed by a long tail. This characteristic is evident in the empirical sales data (triangles) fitted by the solid line. Investigations of multiple generations also suggested that their unit sales exhibit a nearly equivalent initial growth phase [37]. This is also the case here. It has its origin in an almost equivalent competitive advantage of the generations f_{ij}= 1.4 ±0.2 (see Table 2).

A main result of this model is that the evolution of successive generations can be interpreted as an evolutionary adaptation process. The adaptation takes place by a preferential growth of the generation with the higher fitness. As mentioned in the introduction, this was already meant by Victor and Ausubel when they compared the DRAM evolution with that of fruit flies [22]. The technological substitution process becomes a sequence of replacements of generations governed by a Fisher-Pry law, where the fitness advantage is essentially determined by the preference for a generation and the financial success (contained in the reproduction parameter.

For DRAMs the replacement evolution is accompanied by a decrease of the costs per memory unit caused by economies of scale (Moore’s law). Moore’s law predicts an exponential increase of DRAM memory with time respectively a decline of the costs per memory unit. However, the origin of Moore’s law is the application of the lithographic method. Since this technology has a physical boundary the evolutionary adaptation process of DRAMs must be limited and Moore’s law must slow down with time (discussed in [38]).

The diversity of goods we use in our daily life is a consequence of the presented evolutionary adaptation process in a free market. The presented approach suggests that potential buyers decide with their preferences whether a product version of a good replaces previous versions and governs the market for a long time or disappear very soon. This result is in agreement with previous marketing research [14]. If the fitness advantage can be determined with sufficient accuracy the model relations allow the forecast of the unit sales and price evolution of a generation.

5. CONCLUSIONS

The presented dynamic microeconomic model predicts the price and sales dynamics of heterogeneous goods composed of homogenous product generations. The model suggests that:

1. The short term price dispersion of homogeneous generations can be described by a Laplacian.
2. The mean price of a generation declines for a market with fast growing supply according to a logistic law.
3. Product generations of a heterogeneous good are in mutual competition. Their market shares are governed by a Fisher-Pry law.
4. The total unit sales of a heterogeneous good can be described by the life-cycle concept consisting of first- and repurchase of the good.
5. The unit sales of a generation can be given by the product of the total unit sales of the heterogeneous good with the market share of the corresponding generation. It leads to a characteristic sales peak consisting of a steep rise followed by a long tail
6. The presented approach shows that the long term market evolution of successive generations can be understood as an evolutionary adaptation process.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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