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On the Spacelike Curves of R_1^4

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Authors' contributions

This work was carried out in collaboration between both authors. Author MAA designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Authors AIS and MAA managed the analyses of the study and literature searches. Both authors read and approved the final manuscript.

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Abstract

In this paper, we investigate the position vectors of spacelike curves in terms of their curvature functions and give some characterizations for these curves to lie on some subspaces of R_1^4.

Keywords: Spacelike curve; Frenet frame; Minkowski spacetime.

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1 Introduction

Lately, the differential geometry of some special curves in Lorentzian spaces were studied by several authors. Especially the position vectors of spacelike curves, timelike curves and null curves in Lorentzian space were studied by the following authors:

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Null helices in Lorentzian space forms were studied by A. Fernandez, A. Gimenez and P. Lucas [1]. C. Coken and U. Ciftci studied null curves in the 4-dimensional Minkowski space R_1^4 and gave some results for pseudo spherical null curves and Bertrand null curves in [2].

A. T. Ali and M. Onder characterized rectifying spacelike curves in terms of their curvature functions in Minkowski spacetime [3].

K. Ilarslan and O. Boyacioglu studied position vectors of a timelike and a null helice in R_1^3 [4]. K. Ilarslan and E. Nesovic gave some characterizations for null curves in E^4 [5]. K. Ilarslan, E. Nesovic and M. Petrovic-Torgasev studied rectifying curves in Minkowski 3-space [6].

K. Ilarslan studied spacelike normal curves in Minkowski space E_1^3 and gave some characterizations for spacelike normal curves with spacelike, timelike and null principal normal [7]. K. Ilarslan and E. Nesovic studied null osculating curves and gave some characterizations for these curves in Minkowski space-time [8].

M. A. Akgun and A. I. Sivridag studied null Cartan curves in Minkowski 4-space and gave some theorems for these curves to lie on some subspaces of R_1^4 [9]. The authors studied spacelike and timelike curves to lie on some subspaces of R_1^4 and gave some theorems in [10] and [11].

This paper is organized by following: In section 2 we give some basic knowledge related with curves in Minkowski space-time. Section 3 is the original part of this paper. In this section we investigate the conditions for spacelike curves to lie on some subspaces of R_1^4 and we give some characterizations and theorems for these curves.

2 Materials and Methods

Let R_1^4 denote 4-dimensional Minkowski space with a flat Lorentz metric $\langle , \rangle$ given by

$$\langle X, Y\rangle = -x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4$$

where $X = (x_1, x_2, x_3, x_4) \in R_1^4$ and $Y = (y_1, y_2, y_3, y_4) \in R_1^4$. A vector $X \in R_1^4$ is said to be timelike if $\langle X, X\rangle < 0$, spacelike if $\langle X, X\rangle > 0$ and null if $\langle X, X\rangle = 0$ and $X \neq 0$. The norm of a vector $X \in R_1^4$ is denoted by $\|X\|$ and defined by $\|X\| = \sqrt{|\langle X, X\rangle|}$. The vector X is said to be orthogonal to the vector Y if $\langle X, Y\rangle = 0$.

An arbitrary curve $\alpha = \alpha(s)$ in R_1^4 can locally be spacelike, timelike or null, if its velocity vectors $\alpha'(s)$ are respectively spacelike, timelike or null [12].

Let α be a spacelike curve in R_1^4 with the Frenet frame $\{T, N, B_1, B_2\}$ where N and B_2 be null vectors. The Frenet frame $\{T, N, B_1, B_2\}$ has the following conditions:

$$\begin{aligned} \nabla_T T &= k_1 N \\ \nabla_T N &= k_2 B_1 \\ \nabla_T B_1 &= k_3 N - k_2 B_2 \\ \nabla_T B_2 &= -k_1 T - k_3 B_1 \end{aligned} \tag{2.1}$$

where T, N, B_1 and B_2 are mutually orthogonal vectors satisfying the equations

$$\langle B_1, B_1\rangle = \langle T, T\rangle = \langle N, B_2\rangle = 1, \quad \langle N, N\rangle = \langle B_2, B_2\rangle = 0$$

[13].

3 On the Spacelike Curves of R_1^4

In this section we will give some characterizations of spacelike curves to lie on some subspaces of R_1^4. Let α be a spacelike curve in R_1^4 with the Frenet frame $\{T, N, B_1, B_2\}$.

Case 1) First we will investigate the conditions under which the spacelike curve α lies on the subspace spanned by $\{T, N\}$. In this case we can write

$$\alpha(s) = \lambda(s)T + \mu(s)N \tag{3.1}$$

for some differentiable functions λ and μ of the parameter s. Differentiating (3.1) with respect to s and by using the Frenet equations (2.1) we find that

$$\alpha'(s) = \lambda'(s)T + (\lambda(s)k_1(s) + \mu'(s))N + \mu(s)k_2(s)B_1 \tag{3.2}$$

where $\alpha' = T$. Since $\{T, N, B_1, B_2\}$ is a Frenet frame we have the following equations.

$$\begin{cases} \lambda'(s) = 1 \\ \lambda(s)k_1(s) + \mu'(s) = 0 \\ \mu(s)k_2(s) = 0. \end{cases} \tag{3.3}$$

If $\mu(s) = 0$ we obtain $k_1(s) = 0$ and $\lambda(s) = s + c$. So we have

$$\alpha(s) = (s + c)T. \tag{3.4}$$

If $k_2(s) = 0$ we have

$$\mu(s) = -\int (s + c)k_1(s)ds. \tag{3.5}$$

So we have

$$\alpha(s) = (s + c)T - (\int (s + c)k_1(s)ds)N. \tag{3.6}$$

Thus we have the following theorem.

Theorem 3.1. *A spacelike curve α in R_1^4 lies on the subspace spanned by $\{T, N\}$ if and only if it is in the form*

$$\alpha(s) = (s + c)T$$

where $k_1(s) = 0$ or

$$\alpha(s) = (s + c)T - (\int (s + c)k_1(s)ds)N$$

where $k_2(s) = 0$.

Case 2) We will investigate the conditions under which the spacelike curve α lies on the subspace spanned by $\{T, B_1\}$. In this case we can write

$$\alpha(s) = \lambda(s)T + \mu(s)B_1 \tag{3.7}$$

for some differentiable functions λ and μ. Differentiating (3.7) with respect to s and by using the Frenet equations (2.1) we find that

$$\alpha'(s) = \lambda'(s)T + (\lambda(s)k_1(s) + \mu(s)k_3(s))N + \mu'(s)B_1 - \mu(s)k_2(s)B_2. \tag{3.8}$$

Since $\{T, N, B_1, B_2\}$ is a Frenet frame we have the following equations.

$$\begin{cases} \lambda'(s) = 1 \\ \lambda(s)k_1(s) + \mu(s)k_3(s) = 0 \\ \mu(s)k_2(s) = 0 \\ \mu'(s) = 0. \end{cases} \tag{3.9}$$

From (3.9) if $\mu(s) = 0$ then we can write $\lambda(s) = s + c$. So we have

$$\alpha(s) = (s + c)T. \tag{3.10}$$

If $k_2(s) = 0$ we obtain $\lambda(s) = s + c$ and $\mu(s) = -\frac{(s+c)k_1(s)}{k_3(s)}(s + c) = const.$ So we have

$$\alpha(s) = (s + c)T - (\frac{(s + c)k_1(s)}{k_3(s)}(s + c))B_1. \tag{3.11}$$

Thus we have the following theorem.

Theorem 3.2. *A spacelike curve α in R_1^4 lies on the subspace spanned by $\{T, B_1\}$ if and only if it is in the form*

$$\alpha(s) = (s + c)T$$

where $k_1(s) = 0$ or

$$\alpha(s) = (s + c)T - (\frac{(s + c)k_1(s)}{k_3(s)}(s + c))B_1$$

where $k_2(s) = 0$.

Case 3) We will investigate the conditions under which the spacelike curve α lies on the subspace spanned by $\{T, B_2\}$. In this case we can write

$$\alpha(s) = \lambda(s)T + \mu(s)B_2, \tag{3.12}$$

for some differentiable functions λ and μ of the parameter s. Differentiating (3.12) with respect to s and by using the Frenet equations (2.1) we find that

$$\alpha'(s) = (\lambda'(s) - \mu(s)k_1(s))T + \lambda(s)k_1(s)N - \mu(s)k_3(s)B_1 + \mu'(s)B_2. \tag{3.13}$$

Since $\{T, N, B_1, B_2\}$ is a Frenet frame we have the following equations.

$$\begin{cases} \lambda'(s) - \mu(s)k_1(s) = 1 \\ \lambda(s)k_1(s) = 0 \\ \mu(s)k_3(s) = 0 \\ \mu'(s) = 0. \end{cases} \tag{3.14}$$

From (3.14) if $\lambda(s) = 0$ then we obtain $\mu(s) = -\frac{1}{k_1(s)} = const.$ and $k_3(s) = 0$. So we have

$$\alpha(s) = (-\frac{1}{k_1(s)})B_2. \tag{3.15}$$

If $k_1(s) = 0$ then we obtain $\lambda(s) = s + c_1$ and $\mu(s) = c_2$. So we have

$$\alpha(s) = (s + c_1)T + c_2B_2. \tag{3.16}$$

Thus we have the following theorem.

Theorem 3.3. *A spacelike curve α in R_1^4 lies on the subspace spanned by $\{T, B_2\}$ if and only if it is in the form*

$$\alpha(s) = (-\frac{1}{k_1(s)})B_2$$

where $k_3(s) = 0$ *or*

$$\alpha(s) = (s + c_1)T + c_2 B_2$$

where $k_1(s) = 0$.

Case 4) We will investigate the conditions under which the spacelike curve α lies on the subspace spanned by $\{N, B_1\}$. In this case we can write

$$\alpha(s) = \lambda(s)N + \mu(s)B_1 \tag{3.17}$$

for some differentiable functions λ and μ of the parameter s. Differentiating (3.17) with respect to s and by using the Frenet equations (2.1) we find that

$$\alpha'(s) = (\lambda(s) + \mu(s)k_3(s))N + (\lambda(s)k_2(s) + \mu'(s))B_1 - \mu(s)k_2(s)B_2. \tag{3.18}$$

Since α is a spacelike curve $\langle \alpha', \alpha' \rangle = 1$. But from (3.18) we see that $\langle \alpha', \alpha' \rangle = 0$. This is a conradiction. Thus we have the following theorem.

Theorem 3.4. *A spacelike curve α in R_1^4 does not lie on the subspace spanned by $\{N, B_1\}$*

Case 5) We will investigate the conditions under which the spacelike curve α lies on the subspace spanned by $\{N, B_2\}$. In this case we can write

$$\alpha(s) = \lambda(s)N + \mu(s)B_2 \tag{3.19}$$

for some differentiable functions λ and μ of the parameter s. Differentiating (3.19) with respect to s and by using the Frenet equations (2.1) we find that

$$\alpha'(s) = -\mu(s)k_1(s)T + \lambda'(s)N + (\lambda(s)k_2(s) - \mu(s)k_3(s))B_1 + \mu'(s)B_2. \tag{3.20}$$

Since $\{T, N, B_1, B_2\}$ is a Frenet frame we have the following equations.

$$\begin{cases} -\mu(s)k_1(s) = 1 \\ \lambda'(s) = 0 \\ \lambda(s)k_2(s) - \mu(s)k_3(s) = 0 \\ \mu'(s) = 0. \end{cases} \tag{3.21}$$

From (3.21) we can write $\mu(s) = -\frac{1}{k_1(s)} = cons$ and $\lambda(s) = -\frac{k_3(s)}{k_1(s)k_2(s)}$. So we have

$$\alpha(s) = (-\frac{k_3(s)}{k_1(s)k_2(s)})N - \frac{1}{k_1(s)}B_2. \tag{3.22}$$

Thus we have the following theorem.

Theorem 3.5. *A spacelike curve α in R_1^4 lies on the subspace spanned by $\{N, B_2\}$ if and only if it is in the form*

$$\alpha(s) = (-\frac{k_3(s)}{k_1(s)k_2(s)})N - \frac{1}{k_1(s)}B_2.$$

Case 6) We will investigate the conditions under which the spacelike curve α lies on the subspace spanned by $\{B_1, B_2\}$. In this case we can write

$$\alpha(s) = \lambda(s)B_1 + \mu(s)B_2 \tag{3.23}$$

for some differentiable functions λ and μ of the parameter s. Differentiating (3.23) with respect to s and by using the Frenet equations (2.1) we find that

$$\alpha'(s) = -\mu(s)k_1(s)T + \lambda(s)k_3(s)N + (\lambda'(s) - \mu(s)k_3(s))B_1 + (\mu'(s) - \lambda(s)k_2(s))B_2. \tag{3.24}$$

Since $\{T, N, B_1, B_2\}$ is a Frenet frame we have the following equations.

$$\begin{cases} -\mu(s)k_1(s) = 1 \\ \lambda(s)k_3(s) = 0 \\ \lambda'(s) - \mu(s)k_3(s) = 0 \\ \mu'(s) - \lambda(s)k_2(s) = 0. \end{cases} \tag{3.25}$$

If $\lambda(s) = 0$ then we can write $\mu(s) = -\frac{1}{k_1(s)} = cons$. So we have

$$\alpha(s) = -\frac{1}{k_1(s)}B_2. \tag{3.26}$$

If $k_3(s) = 0$ then $\lambda(s) = \frac{k_1'(s)}{k_1^2(s)k_2(s)}$. So we have

$$\alpha(s) = \frac{k_1'(s)}{k_1^2(s)k_2(s)}B_1 - \frac{1}{k_1(s)}B_2. \tag{3.27}$$

Theorem 3.6. *A spacelike curve α in R_1^4 lies on the subspace spanned by $\{B_1, B_2\}$ if and only if it is in the form*

$$\alpha(s) = -\frac{1}{k_1(s)}B_2$$

or

$$\alpha(s) = \frac{k_1'(s)}{k_1^2(s)k_2(s)}B_1 - \frac{1}{k_1(s)}B_2$$

where $k_3(s) = 0$

Case 7) We will investigate the conditions under which the spacelike curve α lies on the subspace spanned by $\{T, N, B_1\}$. In this case we can write

$$\alpha(s) = \lambda(s)T + \mu(s)N + \gamma(s)B_1 \tag{3.28}$$

for some differentiable functions λ, μ and γ of the parameter s. Differentiating (3.28) with respect to s and by using the Frenet equations (2.1) we find that

$$\begin{aligned} \alpha'(s) &= \lambda'(s)T + (\lambda(s)k_1(s) + \mu'(s) + \gamma(s)k_3(s))N \\ &+ (\mu(s)k_2(s) + \gamma'(s))B_1 - \gamma(s)k_2(s)B_2. \end{aligned} \tag{3.29}$$

Since $\{T, N, B_1, B_2\}$ is a Frenet frame we have the following equations.

$$\begin{cases} \lambda'(s) = 1 \\ \lambda(s)k_1(s) + \mu'(s) + \gamma(s)k_3(s) = 0 \\ \mu(s)k_2(s) + \gamma'(s) = 0 \\ \gamma(s)k_2(s) = 0. \end{cases} \tag{3.30}$$

From the equation $\gamma(s)k_2(s) = 0$ if $\gamma(s) = 0$ we see that $\mu(s)k_2(s) = 0$. If $\mu(s) = 0$ then we can write $\lambda(s) = s + c$ and $k_1(s) = 0$. So we have

$$\alpha(s) = (s + c)T. \tag{3.31}$$

If $\gamma(s) = 0$ and $k_2(s) = 0$ then we can write $\mu(s) = -\int(s + c))k_1(s)ds$. So we have

$$\alpha(s) = (s + c)T - (\int(s + c))k_1(s)ds)N. \tag{3.32}$$

If $\gamma(s) \neq 0$ and $k_2(s) = 0$ then we can write $\lambda(s) = s + c$, $\gamma(s) = c_1$ and $\mu(s) = -\int(c_1k_3(s) + (s + c)k_1(s))ds$. So we have

$$\alpha(s) = (s + c)T - (\int(c_1k_3(s) + (s + c)k_1(s))ds)N + c_1B_1. \tag{3.33}$$

Thus we have the following theorem.

Theorem 3.7. *A spacelike curve α in R_1^4 lies on the subspace spanned by $\{T, N, B_1\}$ if and only if it is in the form*

$$\alpha(s) = (s + c)T$$

where $k_1(s) = 0$ or

$$\alpha(s) = (s + c)T - (\int(s + c))k_1(s)ds)N$$

or

$$\alpha(s) = (s + c)T - (\int(c_1k_3(s) + (s + c)k_1(s))ds)N + c_1B_1$$

where $k_2(s) = 0$ and c, c_1 are constants.

Case 8) We will investigate the conditions under which the spacelike curve α lies on the subspace spanned by $\{T, N, B_2\}$. In this case we can write

$$\alpha(s) = \lambda(s)T + \mu(s)N + \gamma(s)B_2 \tag{3.34}$$

for some differentiable functions λ, μ and γ of the parameter s. Differentiating (3.34) with respect to s and by using the Frenet equations (2.1) we find that

$$\alpha'(s) = (\lambda'(s) - \gamma(s)k_1(s))T + (\lambda(s)k_1(s) + \mu'(s))N + (\mu(s)k_2(s) - \gamma(s)k_3(s))B_1 + \gamma'(s)B_2. \tag{3.35}$$

Since $\{T, N, B_1, B_2\}$ is a Frenet frame we have the following equations:

$$\begin{cases} \lambda'(s) - \gamma(s)k_1(s) = 1 \\ \lambda(s)k_1(s) + \mu'(s) = 0 \\ \mu(s)k_2(s) - \gamma(s)k_3(s) = 0 \\ \gamma'(s) = 0. \end{cases} \tag{3.36}$$

From (3.36) we find $\gamma(s) = c$ and $\mu(s) = c\frac{k_3(s)}{k_2(s)}$. If we use these equations in (3.36) we obtain

$$\lambda(s) = c\frac{k_3(s)k_2'(s) - k_3'(s)k_2(s)}{k_1^2(s)k_2(s)}. \tag{3.37}$$

So we have

$$\alpha(s) = (c\frac{k_3(s)k_2'(s) - k_3'(s)k_2(s)}{k_1^2(s)k_2(s)})T + c\frac{k_3(s)}{k_2(s)}N + cB_2. \tag{3.38}$$

Thus we have the following theorem.

Theorem 3.8. *A spacelike curve α in R^4_1 lies on the subspace spanned by $\{T, N, B_2\}$ if and only if it is in the form*

$$\alpha(s) = (c\frac{k_3(s)k_2'(s) - k_3'(s)k_2(s)}{k_1^2(s)k_2(s)})T + c\frac{k_3(s)}{k_2(s)}N + cB_2.$$

Case 9) We will investigate the conditions under which the spacelike curve α lies on the subspace spanned by $\{T, B_1, B_2\}$. In this case we can write

$$\alpha(s) = \lambda(s)T + \mu(s)B_1 + \gamma(s)B_2 \tag{3.39}$$

for some differentiable functions λ, μ and γ of the parameter s. Differentiating (3.39) with respect to s and by using the Frenet equations (2.1) we find that

$$\begin{aligned} \alpha'(s) &= (\lambda'(s) - \gamma(s)k_1(s))T + (\lambda(s)k_1(s) + \mu(s)k_3(s))N + (\mu'(s) - \gamma(s)k_3(s))B_1 \\ &+ (\gamma'(s) - \mu(s)k_2(s))B_2. \end{aligned} \tag{3.40}$$

Since $\{T, N, B_1, B_2\}$ is a Frenet frame we have the following equations.

$$\begin{cases} \lambda'(s) - \gamma(s)k_1(s) = 1 \\ \lambda(s)k_1(s) + \mu(s)k_3(s) = 0 \\ \mu'(s) - \gamma(s)k_3(s) = 0 \\ \gamma'(s) - \mu(s)k_2(s) = 0. \end{cases} \tag{3.41}$$

From (3.41) we can write the differential equation

$$\frac{d}{ds}(\frac{k_3(s)}{k_1(s)}\mu(s)) + \frac{k_1(s)}{k_3(s)}\frac{d\mu(s)}{ds} = -1. \tag{3.42}$$

By using exchange variable $t = \int_0^s \frac{k_3(s)}{k_1(s)}ds$ in (3.42) we find

$$2\frac{d\mu(s)}{ds} = -1. \tag{3.43}$$

From the solution of this equation we find

$$\mu(s) = -\frac{t}{2} + c. \tag{3.44}$$

Replacing variable $t = \int_0^s \frac{k_3(s)}{k_1(s)}ds$ in (3.44) we find

$$\mu(s) = -\frac{1}{2}\int \frac{k_3(s)}{k_1(s)}ds + c. \tag{3.45}$$

From (3.41) and (3.45) we find

$$\lambda(s) = \frac{1}{2}\frac{k_3(s)}{k_1(s)}\int \frac{k_3(s)}{k_1(s)}ds - c\frac{k_3(s)}{k_1(s)}. \tag{3.46}$$

From (3.41), (3.45) and (3.46) we find

$$\gamma(s) = -\frac{1}{2k_1(s)}. \tag{3.47}$$

So we have

$$\alpha(s) = (\frac{1}{2}\frac{k_3(s)}{k_1(s)}\int \frac{k_3(s)}{k_1(s)}ds - c\frac{k_3(s)}{k_1(s)})T + (-\frac{1}{2}\int \frac{k_3(s)}{k_1(s)}ds + c)B_1 - (\frac{1}{2k_1(s)})B_2. \tag{3.48}$$

Thus we have the following theorem.

Theorem 3.9. *A spacelike curve α in R_1^4 lies on the subspace spanned by $\{T, B_1, B_2\}$ if and only if it is in the form*

$$\alpha(s) = (\frac{1}{2}\frac{k_3(s)}{k_1(s)}\int \frac{k_3(s)}{k_1(s)}ds - c\frac{k_3(s)}{k_1(s)})T + (-\frac{1}{2}\int \frac{k_3(s)}{k_1(s)}ds + c)B_1 - (\frac{1}{2k_1(s)})B_2.$$

Case 10) We will investigate the conditions under which the spacelike curve α lies on the subspace spanned by $\{N, B_1, B_2\}$. In this case we can write

$$\alpha(s) = \lambda(s)N + \mu(s)B_1 + \gamma(s)B_2 \tag{3.49}$$

for some differentiable functions λ, μ and γ of the parameter s. Differentiating (3.49) with respect to s and by using the Frenet equations (2.1) we find that

$$\begin{aligned} \alpha'(s) &= -\gamma(s)k_1(s)T + (\lambda'(s) + \mu(s)k_3(s))N + (\lambda(s)k_2(s) + \mu'(s) - \gamma(s)k_3(s))B_1 \\ &+ (\gamma'(s) - \mu(s)k_2(s))B_2. \end{aligned} \tag{3.50}$$

Since $\{T, N, B_1, B_2\}$ is a Frenet frame we have the following equations.

$$\begin{cases} -\gamma(s)k_1(s) = 1 \\ \lambda'(s) + \mu(s)k_3(s) = 0 \\ \lambda(s)k_2(s) + \mu'(s) - \gamma(s)k_3(s) = 0 \\ \gamma'(s) - \mu(s)k_2(s) = 0. \end{cases} \tag{3.51}$$

From (3.51) we can write $\gamma(s) = -\frac{1}{k_1(s)}$. If we use the last equation in $\gamma'(s) - \mu(s)k_2(s) = 0$ we obtain $\mu(s) = \frac{k_1'(s)}{k_1^2(s)k_2(s)}$. From (3.51) we find

$$\lambda(s) = -\frac{k_3(s)}{k_1(s)k_2(s)} - \frac{k_1''(s)k_1(s)k_2(s) - k_1'(s)(2k_1'(s)k_2(s) + k_1(s)k_2'(s))}{k_1^3(s)k_2^2(s)}. \tag{3.52}$$

So we have

$$\begin{aligned} \alpha(s) &= (-\frac{k_3(s)}{k_1(s)k_2(s)} - \frac{k_1''(s)k_1(s)k_2(s) - k_1'(s)(2k_1'(s)k_2(s) + k_1(s)k_2'(s))}{k_1^3(s)k_2^2(s)})N \\ &+ (\frac{k_1'(s)}{k_1^2(s)k_2(s)})B_1 - (\frac{1}{k_1(s)})B_2. \end{aligned} \tag{3.53}$$

Thus we have the following theorem.

Theorem 3.10. *A spacelike curve α in R_1^4 lies on the subspace spanned by $\{N, B_1, B_2\}$ if and only if it is in the form*

$$\begin{aligned} \alpha(s) &= (-\frac{k_3(s)}{k_1(s)k_2(s)} - \frac{k_1''(s)k_1(s)k_2(s) - k_1'(s)(2k_1'(s)k_2(s) + k_1(s)k_2'(s))}{k_1^3(s)k_2^2(s)})N \\ &+ (\frac{k_1'(s)}{k_1^2(s)k_2(s)})B_1 - (\frac{1}{k_1(s)})B_2. \end{aligned}$$

4 Conclusions

a In the section (2), we defined basic terminologies needed to read this paper.

b In the section (3), we characterize the spacelike curves in term of their curvature functions in R_1^4. So we see that spacelike curves lie on all the supspaces of R_1^4 excluding the subspace spanned by $\{N, B_1\}$.

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Competing Interests

Authors have declared that no competing interests exist.

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