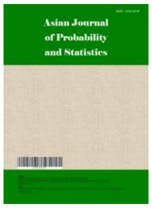

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Difference-Cum-Ratio Estimators for Estimating Finite Population Coefficient of Variation in Simple Random Sampling

A. Audu[1], M. A. Yunusa[1], O. O. Ishaq[2*], M. K. Lawal[3], A. Rashida[4], A. H. Muhammad[4], A. B. Bello[4], M. U. Hairullahi[4] and J. O. Muili[1]

[1]Department of Mathematics, Usmanu Danfodiyo University Sokoto, Nigeria.
[2]Department of Statistics, Kano University of Science and Technology, Nigeria.
[3]Academic Planning Unit, Federal Polytechnic Bida, Nigeria.
[4]State College of Basic and Remedial Studies, Sokoto, Nigeria.

Authors' contributions

This work was carried out in collaboration among all authors. Authors AA, MAY and OOI designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors AR and JOM managed the analyses of the study. Authors MKL, AHM, ABB and MUH managed the literature searches. All authors read and approved the final manuscript.

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Abstract

In this paper, three difference-cum-ratio estimators for estimating finite population coefficient of variation of the study variable using known population mean, population variance and population coefficient of variation of auxiliary variable were suggested. The biases and mean square errors (MSEs) of the proposed estimators were obtained. The relative performance of the proposed estimators with respect to that of some existing estimators were assessed using two populations' information. The results showed that the proposed estimators were more efficient than the usual unbiased, ratio type, exponential ratio-type, difference-type and other existing estimators considered in the study.

Keywords: Auxiliary variable; MSE; coefficient of variation; study variable; simple random sampling.

**Corresponding author: E-mail: babington4u@gmail.com, ishaq.o.o@kustwudil.edu.ng;*

1 Introduction

Several studies in the theory of sampling survey have established the fact that the use of auxiliary information at the planning and estimation stages helps in enhancing the efficiency of estimators for estimating population parameters like population mean, population variance, standard deviation etc. as compared to the estimators which use not auxiliary information. Several authors like Singh HP et al,[1], Sahai A et al. [2], Srivastava Sk et al. [3], Ahmed A et al. [4], Audu A et al. [5], Audu A et al. [6], Muili JO et al.[7] have worked extensively in that direction. Authors like Singh HP et al.[8], Sisodia BVS et al. [9], Khoshnevisan M et al. [10], Singh RVK et al. [11], Ahmed A et al.[12] and Audu A et al.[13] utilized coefficient of variation of auxiliary variable in the estimators formulation and obtanied highly efficient estimators. Nevertheless the investigators did not emphasized on the problem of estimation of coefficient of variation. The coefficient of variation is one of the major parameters of population often used in comparing variability measured in different units. The coefficient of variation is expressed in percentages to indicate the extent of variability percent in the data. Some authors have worked on this, for instance [8,9] first proposed the estimator for coefficient of variation when samples were selected using SRSWOR. Das AK et al. [14], Das AK et al.[15], Patel PA et al. [16], Rajyaguru A et al. [17], Rajyaguru A et al. [18], Archana V et al. [19], Singh R et al. [20] also, worked on the problem of estimation of coefficient of variation (C.V) under simple random sampling and stratified random sampling. However, some of these existing estimators are ratio-based which are less efficient if the correlation between the study and auxiliary variables is weak. To address this flaw, concept of difference estimator which does not necessary required strong correlation was used to obtained new estimators.

In the present study, we have considered the problem of estimation of population coefficient of variation utilizing information on a single auxiliary variable in SRSWOR by proposing difference-cum-ratio estimators for estimating the population coefficient of variation of the study variable. These new estimators are expected to give a precise and efficient estimate of the population coefficient of variation of the study variable than existing estimators considered in the study.

2 Some Existing Estimators in Literature

The usual unbiased estimator using information on single auxiliary variable to estimate the population coefficient of variation is given by:

$$t_0 = \hat{C}_y = \frac{s_y}{\bar{y}} \tag{1}$$

where, $\bar{y} = n^{-1}\sum_{i=1}^{n} y_i$ and $s_y^2 = (n-1)^{-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ are the sample mean and variance respectively.

The mean square error (MSE) expression of the estimator t_0 is given by:

$$MSE(t_0) = C_y^2\gamma\left(C_y^2 + 0.25(\lambda_{40} - 1) - C_y\lambda_{30}\right) \tag{2}$$

where $\gamma = n^{-1} - N^{-1}$, $\lambda_{rs} = \dfrac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$, $\mu_{rs} = \dfrac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^r(x_i - \bar{x})^s$, $\lambda_{rs} = \dfrac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$.

Archana V et al. [19] proposed ratio estimators of population coefficient of variation under simple random sampling without replacement (SRSWOR) as:

$$t_{AR} = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right) \tag{3}$$

Where, $\bar{x} = n^{-1}\sum_{i=1}^{n} x_i$ and $\bar{X} = N^{-1}\sum_{i=1}^{N} X_i$ are the sample and population mean of the auxiliary variable respectively.

The mean square error (MSE) expression of the estimator t_{AR} is given by:

$$MSE\left(t_{AR}\right) = C_y^2\gamma\left[C_y^2 + 0.25\left(\lambda_{40}-1\right) - C_y\lambda_{30} - C_x\lambda_{21} + 0.25\left(\lambda_{40}-1\right)\right] \tag{4}$$

Singh R et al. [20] proposed ratio-type, exponential ratio-type and difference-type estimators for coefficient of variation of the study variable Y using mean of auxiliary variable and are given below with their MSEs as

$$t_1 = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha} \tag{5}$$

$$t_2 = \hat{C}_y \exp\left\{\beta\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right\} \tag{6}$$

$$t_3 = \hat{C}_y + d_1\left(\bar{X}-\bar{x}\right) \tag{7}$$

$$MSE(t_1) = C_y^2\gamma\left[C_y^2 + \frac{\lambda_{40}-1}{4} + \alpha^2 C_x^2 - C_y\lambda_{30} + 2\rho C_y C_x - \alpha C_x\lambda_{21}\right] \tag{8}$$

$$MSE(t_2) = C_y^2\gamma\left[C_y^2 + \frac{\lambda_{40}-1}{4} + \frac{\beta^2 C_x^2}{4} - C_y\lambda_{30} + \beta\rho C_y C_x - \frac{\beta}{2}C_x\lambda_{21}\right] \tag{9}$$

$$MSE(t_3) = \gamma\left[C_y^2\left(C_y^2 - C_y\lambda_{30} + \frac{\lambda_{40}-1}{4}\right) + d_1^2\bar{X}^2C_x^2 + 2d_1\bar{X}\rho C_y^2 C_x - d_1\bar{X}C_y C_x\lambda_{21}\right] \tag{10}$$

Where $\alpha = \dfrac{\lambda_{21}-2\rho C_y}{2C_x}$, $\beta = \dfrac{\lambda_{21}-2\rho C_y}{C_x}$, $d_1 = \dfrac{C_y\lambda_{21}-2\rho C_y^2}{2\bar{X}C_x}$..

Singh R et al. [20] proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_0 and $t_1$estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_4^{AM} = \frac{\hat{C}_y}{2}\left[1+\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha}\right] \tag{11}$$

$$t_4^{GM} = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right)^{\frac{\alpha}{2}} \tag{12}$$

$$t_4^{HM} = 2\hat{C}_y \left[1 + \left(\frac{\bar{x}}{\bar{X}} \right)^{\alpha} \right]^{-1} \tag{13}$$

$$MSE(t_4^i) = C_y^2 \gamma \left[C_y^2 + \frac{\lambda_{40} - 1}{4} + \frac{\alpha^2 C_x^2}{4} - C_y \lambda_{30} + \alpha \rho C_y C_x - \frac{\alpha}{2} C_x \lambda_{21} \right] \tag{14}$$

Where $\alpha = \dfrac{\lambda_{21} - 2\rho C_y}{C_x}$, $j = AM, GM, HM$.

Singh R et al. [20] proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_0 and t_2 estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_5^{AM} = \frac{\hat{C}_y}{2} \left[1 + \exp\left\{ \beta \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \right\} \right] \tag{15}$$

$$t_5^{GM} = \hat{C}_y \exp\left\{ \frac{\beta}{2} \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \right\} \tag{16}$$

$$t_5^{HM} = 2\hat{C}_y \left[1 + \exp\left\{ -\beta \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \right\} \right]^{-1} \tag{17}$$

$$MSE(t_5^i) = C_y^2 \gamma \left[C_y^2 + \frac{\lambda_{40} - 1}{4} + \frac{\beta^2 C_x^2}{16} - C_y \lambda_{30} + \frac{\beta}{2} \rho C_y C_x - \frac{\beta}{4} C_x \lambda_{21} \right] \tag{18}$$

where $\beta = \dfrac{2\left(\lambda_{21} - 2\rho C_y\right)}{C_x}$.

Singh R et al. [20] proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_1 and t_2 estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_6^{AM} = \frac{\hat{C}_y}{2} \left[\left(\frac{\bar{X}}{\bar{x}} \right)^{\alpha} + \exp\left\{ \beta \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \right\} \right] \tag{19}$$

$$t_6^{GM} = \hat{C}_y \left(\frac{\bar{X}}{\bar{x}} \right)^{\frac{\alpha}{2}} \exp\left\{ \frac{\beta}{2} \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \right\} \tag{20}$$

$$t_6^{HM} = 2\hat{C}_y \left[\left(\frac{\bar{x}}{\bar{X}} \right)^{\alpha} + \exp\left\{ -\beta \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \right\} \right]^{-1} \tag{21}$$

$$MSE(t_6^i) = C_y^2 \gamma \left[\begin{array}{l} C_y^2 + \frac{\lambda_{40} - 1}{4} + \frac{1}{4}\left(\alpha + \frac{\beta}{2} \right)^2 C_x^2 - C_y \lambda_{30} - \frac{1}{2}\left(\alpha + \frac{\beta}{2} \right) C_x \lambda_{21} \\ + \left(\alpha + \frac{\beta}{2} \right) \rho C_y C_x \end{array} \right] \tag{22}$$

where $\beta = 2\left(\frac{\lambda_{21} - 2\rho C_y}{C_x} - \alpha \right)$,

Singh R et al. [20] proposed ratio type, exponential ratio-type and difference-type estimators for estimating coefficient of variation of the study variable Y using variances of the auxiliary variables and are given below:

$$t_7 = \hat{C}_y \left(\frac{S_x^2}{s_x^2} \right)^{\alpha} \tag{23}$$

$$t_8 = \hat{C}_y \exp\left\{ \beta \left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2} \right) \right\} \tag{24}$$

$$t_9 = \hat{C}_y + d_2 \left(S_x^2 - s_x^2 \right) \tag{25}$$

$$MSE(t_7) = C_y^2 \gamma \left[C_y^2 + \frac{\lambda_{40} - 1}{4} + \alpha^2 \left(\lambda_{04} - 1 \right) - C_y \lambda_{30} + 2\alpha C_y \lambda_{12} - \alpha \left(\lambda_{22} - 1 \right) \right] \tag{26}$$

$$MSE(t_8) = C_y^2 \gamma \left[C_y^2 + \frac{\lambda_{40} - 1}{4} + \frac{\beta^2 \left(\lambda_{04} - 1 \right)}{4} - C_y \lambda_{30} + \beta C_y \lambda_{12} - \frac{\beta}{2} \left(\lambda_{22} - 1 \right) \right] \tag{27}$$

$$MSE(t_9) = \gamma \left[\begin{array}{l} C_y^2 \left(C_y^2 - C_y \lambda_{30} + \frac{\lambda_{40} - 1}{4} \right) + 2C_y^2 d_2 S_x^2 \lambda_{12} + d_2^2 S_x^4 \left(\lambda_{04} - 1 \right) \\ - C_y d_2 S_x^2 \left(\lambda_{22} - 1 \right) \end{array} \right] \tag{28}$$

Where $s_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2, \alpha = \frac{\lambda_{22} - 1 - 2C_y \lambda_{12}}{2\left(\lambda_{04} - 1 \right)}$, $\beta = \frac{\lambda_{22} - 1 - 2C_y \lambda_{12}}{\left(\lambda_{04} - 1 \right)}$, $d_2 = \frac{C_y \left(\lambda_{22} - 1 \right) - 2C_y^2 \lambda_{12}}{2S_x^2 \left(\lambda_{04} - 1 \right)}$.

Singh R et al. [20] proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_0 and t_7 estimators for estimating coefficient of variation of the study variable Y and are given below with their

MSEs as

$$t_{10}^{AM} = \frac{\hat{C}_y}{2}\left[1+\left(\frac{S_x^2}{s_x^2}\right)^{\alpha}\right] \tag{29}$$

$$t_{10}^{GM} = \hat{C}_y\left(\frac{S_x^2}{s_x^2}\right)^{\frac{\alpha}{2}} \tag{30}$$

$$t_{10}^{HM} = 2\hat{C}_y\left[1+\left(\frac{s_x^2}{S_x^2}\right)^{\alpha}\right]^{-1} \tag{31}$$

$$MSE(t_{10}^{i}) = C_y^2\gamma\left[C_y^2+\frac{\lambda_{40}-1}{4}+\frac{\alpha^2\left(\lambda_{04}-1\right)}{4}-C_y\lambda_{30}+\alpha C_y\lambda_{12}-\frac{\alpha}{2}\left(\lambda_{22}-1\right)\right] \tag{32}$$

where $\alpha = \dfrac{\lambda_{22}-1-2C_y\lambda_{12}}{\left(\lambda_{04}-1\right)}$.

Rajyaguru A et al. [17] proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_0 and t_8 estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_{11}^{AM} = \frac{\hat{C}_y}{2}\left[1+\exp\left\{\beta\left(\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right)\right\}\right] \tag{33}$$

$$t_{11}^{GM} = \hat{C}_y\exp\left\{\frac{\beta}{2}\left(\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right)\right\} \tag{34}$$

$$t_{11}^{HM} = 2\hat{C}_y\left[1+\exp\left\{-\beta\left(\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right)\right\}\right]^{-1} \tag{35}$$

$$MSE(t_{11}^{i}) = C_y^2\gamma\left[C_y^2+\frac{\lambda_{40}-1}{4}+\frac{\beta^2C_x^2}{16}-C_y\lambda_{30}+\frac{\beta}{2}C_y\lambda_{12}-\frac{\beta}{4}\left(\lambda_{22}-1\right)\right] \tag{36}$$

where $\beta = \dfrac{2\left(\lambda_{22}-1\right)-4C_y\lambda_{12}}{\left(\lambda_{04}-1\right)}$.

Singh R et al. [20] proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_7 and t_8 estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_{12}^{AM} = \frac{\hat{C}_y}{2}\left[\left(\frac{S_x^2}{s_x^2}\right)^{\alpha} + \exp\left\{\beta\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right\}\right] \tag{37}$$

$$t_{12}^{GM} = \hat{C}_y\left(\frac{S_x^2}{s_x^2}\right)^{\frac{\alpha}{2}} \exp\left\{\frac{\beta}{2}\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right\} \tag{38}$$

$$t_{12}^{HM} = 2\hat{C}_y\left[\left(\frac{s_x^2}{S_x^2}\right)^{\alpha} + \exp\left\{-\beta\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right)\right\}\right]^{-1} \tag{39}$$

$$MSE(t_{12}^{i}) = C_y^2\gamma\left[\begin{array}{l} C_y^2 + \frac{\lambda_{40}-1}{4} + \frac{1}{4}\left(\alpha + \frac{\beta}{2}\right)^2(\lambda_{04}-1) - C_y\lambda_{30} - \frac{1}{2}\left(\alpha + \frac{\beta}{2}\right)(\lambda_{22}-1) \\ +\left(\alpha + \frac{\beta}{2}\right)C_y\lambda_{12} \end{array}\right] \tag{40}$$

where $\beta = 2\left(\frac{\lambda_{22} - 1 - 2C_y\lambda_{12}}{(\lambda_{04}-1)} - \alpha\right)$.

Singh R et al. [20] proposed ratio-type, exponential ratio-type and difference-type estimators for estimating coefficient of variation of the study variable Y using coefficients of variation of auxiliary variable, and are given below with their MSEs as

$$t_{13} = \hat{C}_y\left(\frac{C_x}{c_x}\right)^{\alpha} \tag{41}$$

$$t_{14} = \hat{C}_y \exp\left\{\beta\left(\frac{C_x - c_x}{C_x + c_x}\right)\right\} \tag{42}$$

$$t_{15} = \hat{C}_y + d_3(C_x - c_x) \tag{43}$$

$$MSE(t_{13}) = C_y^2\gamma\left[\begin{array}{l} C_y^2 + \frac{\lambda_{40}-1}{4} + \frac{\alpha^2(\lambda_{04}-1)}{4} + \alpha^2C_x^2 - C_y\lambda_{30} - 2\alpha\rho C_yC_x + \alpha C_x\lambda_{21} + \alpha C_y\lambda_{12} \\ -\frac{\alpha(\lambda_{22}-1)}{2} - \alpha^2C_x\lambda_{03} \end{array}\right] \tag{44}$$

$$MSE(t_{14}) = C_y^2\gamma\left[\begin{array}{l} C_y^2 + \frac{\lambda_{40}-1}{4} + \frac{\beta^2C_x^2}{4} - C_y\lambda_{30} - \beta\rho C_yC_x + \frac{\beta}{2}C_x\lambda_{21} + \frac{\beta}{2}C_y\lambda_{12} + \frac{\beta^2(\lambda_{04}-1)}{16} \\ -\frac{\beta(\lambda_{22}-1)}{4} - \frac{\beta^2}{4}C_x\lambda_{03} \end{array}\right] \tag{45}$$

$$MSE(t_{15}) = \gamma \begin{bmatrix} C_y^2\left(C_y^2 - C_y\lambda_{30} + \dfrac{\lambda_{40}-1}{4}\right) + d_3^2 C_x^2\left(C_x^2 + \dfrac{\lambda_{40}-1}{4} - C_x\lambda_{03}\right) \\ +2d_3 C_y C_x\left(\rho C_y C_x - \dfrac{1}{2}\left(C_y\lambda_{12} + C_x\lambda_{21} - (\lambda_{22}-1)\right)\right) \end{bmatrix} \quad (46)$$

Where $\alpha = \dfrac{2\rho C_y C_x + 0.5(\lambda_{22}-1) - C_y\lambda_{12} - C_x\lambda_{21}}{2C_x^2 + 0.5(\lambda_{04}-1) - 2C_x\lambda_{03}}$, $\beta = \dfrac{8\rho C_y C_x + 2(\lambda_{22}-1) - 4C_y\lambda_{12} - 4C_x\lambda_{21}}{4C_x^2 + (\lambda_{04}-1) - 4C_x\lambda_{03}}$,

$d_3 = \dfrac{-C_y\left(\rho C_y C_x - 0.5\left(C_y\lambda_{12} + C_x\lambda_{21}\right) + 0.25(\lambda_{22}-1)\right)}{C_x\left(C_x^2 + 0.25(\lambda_{04}-1) - C_x\lambda_{03}\right)}$.

Singh R et al. [20] proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_0 and t_{13} estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_{16}^{AM} = \frac{\hat{C}_y}{2}\left[1 + \left(\frac{C_x}{c_x}\right)^{\alpha}\right] \quad (47)$$

$$t_{16}^{GM} = \hat{C}_y\left(\frac{C_x}{c_x}\right)^{\frac{\alpha}{2}} \quad (48)$$

$$t_{16}^{HM} = 2\hat{C}_y\left[1 + \left(\frac{c_x}{C_x}\right)^{\alpha}\right]^{-1} \quad (49)$$

$$MSE(t_{16}^{i}) = C_y^2\gamma \begin{bmatrix} C_y^2 + \dfrac{\lambda_{40}-1}{4} + \dfrac{\alpha^2 C_x^2}{4} - C_y\lambda_{30} - \alpha\rho C_y C_x + \dfrac{\alpha}{2}C_x\lambda_{21} + \dfrac{\alpha}{2}C_y\lambda_{12} + \dfrac{\alpha^2(\lambda_{04}-1)}{16} \\ -\dfrac{\alpha(\lambda_{22}-1)}{4} - \dfrac{\alpha^2}{4}C_x\lambda_{03} \end{bmatrix} \quad (50)$$

where $\alpha = \dfrac{8\rho C_y C_x + 2(\lambda_{22}-1) - 4C_y\lambda_{12} - 4C_x\lambda_{21}}{4C_x^2 + (\lambda_{04}-1) - 4C_x\lambda_{03}}$.

Singh R et al. [20] proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_0 and t_{14} estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_{17}^{AM} = \frac{\hat{C}_y}{2}\left[1 + \exp\left\{\beta\left(\frac{C_x - c_x}{C_x + c_x}\right)\right\}\right] \quad (51)$$

$$t_{17}^{GM} = \hat{C}_y \exp\left\{ \frac{\beta}{2} \left(\frac{C_x - c_x}{C_x + c_x} \right) \right\} \tag{52}$$

$$t_{17}^{HM} = 2\hat{C}_y \left[1 + \exp\left\{ -\beta \left(\frac{C_x - c_x}{C_x + c_x} \right) \right\} \right]^{-1} \tag{53}$$

$$MSE(t_{17}^{i}) = C_y^2 \gamma \left[\begin{array}{l} C_y^2 + \frac{\lambda_{40} - 1}{4} + \frac{\beta^2 (\lambda_{04} - 1)}{64} + \frac{\beta^2 C_x^2}{16} - \frac{\beta^2 C_x \lambda_{03}}{16} - C_y \lambda_{30} \\ -\frac{\beta \rho C_y C_x}{2} - \frac{\beta (\lambda_{22} - 1)}{8} + \frac{\beta C_x \lambda_{21}}{4} + \frac{\beta C_y \lambda_{12}}{4} \end{array} \right] \tag{54}$$

where $\beta = \dfrac{16 \rho C_y C_x + 4(\lambda_{22} - 1) - 8 C_y \lambda_{12} - 8 C_x \lambda_{21}}{4 C_x^2 + (\lambda_{04} - 1) - 4 C_x \lambda_{03}}$.

Singh R et al. [20] proposed arithmetic, geometric and harmonic mean estimators (AM, GM, HM) based on t_{13} and t_{15} estimators for estimating coefficient of variation of the study variable Y and are given below with their MSEs as

$$t_{18}^{AM} = \frac{\hat{C}_y}{2} \left[\left(\frac{C_x}{c_x} \right)^{\alpha} + \exp\left\{ \beta \left(\frac{C_x - c_x}{C_x + c_x} \right) \right\} \right] \tag{55}$$

$$t_{18}^{GM} = \hat{C}_y \left(\frac{C_x}{c_x} \right)^{\frac{\alpha}{2}} \exp\left\{ \frac{\beta}{2} \left(\frac{C_x - c_x}{C_x + c_x} \right) \right\} \tag{56}$$

$$t_{18}^{HM} = 2\hat{C}_y \left[\left(\frac{c_x}{C_x} \right)^{\alpha} + \exp\left\{ -\beta \left(\frac{C_x - c_x}{C_x + c_x} \right) \right\} \right]^{-1} \tag{57}$$

$$MSE(t_{18}^{i}) = C_y^2 \gamma \left[\begin{array}{l} C_y^2 + \frac{\lambda_{40} - 1}{4} - C_y \lambda_{30} + \frac{1}{4} \left(\alpha + \frac{\beta}{2} \right)^2 \left(C_x^2 + \frac{\lambda_{04} - 1}{4} - C_x \lambda_{03} \right) \\ + \left(\alpha + \frac{\beta}{2} \right) \left(\frac{C_y \lambda_{12}}{2} + \frac{C_x \lambda_{21}}{2} - \frac{\lambda_{22} - 1}{4} - \rho C_y C_x \right) \end{array} \right] \tag{58}$$

where $\beta = 2 \left[\dfrac{2 \rho C_y C_x - C_y \lambda_{12} - C_x \lambda_{21} + 0.5(\lambda_{22} - 1)}{C_x^2 + 0.25(\lambda_{04} - 1) - C_x \lambda_{03}} - \alpha \right]$.

3 Methodology

3.1 Proposed estimators

Having studied the estimators in section 2, we proposed three difference-cum-ratio estimators for coefficient of variation based on information on a single auxiliary variable in three sections.

$$T_{M1}=\left[\frac{\hat{C}_y}{2}\left(\frac{\bar{X}}{\bar{x}}+\frac{\bar{x}}{\bar{X}}\right)+w_1\left(\bar{X}-\bar{x}\right)+w_2\hat{C}_y\right]\left(\frac{\bar{X}}{\bar{x}}\right) \tag{59}$$

$$T_{M2}=\left[\frac{\hat{C}_y}{2}\left(\frac{S_x^2}{s_x^2}+\frac{s_x^2}{S_x^2}\right)+w_3\left(S_x^2-s_x^2\right)+w_4\hat{C}_y\right]\left(\frac{S_x^2}{s_x^2}\right) \tag{60}$$

$$T_{M3}=\left[\frac{\hat{C}_y}{2}\left(\frac{C_x}{c_x}+\frac{c_x}{C_x}\right)+w_5\left(C_x-c_x\right)+w_6\hat{C}_y\right]\left(\frac{C_x}{c_x}\right) \tag{61}$$

where $w_k, k=1,2,...,6$ are unknown functions to be estimated by minimizing the MSEs of $T_{Mj}, j=1,2,3$

3.2 Properties (Bias and MSE) of $T_{Mj}, j=1,2,3$

Now let us define:

$$e_0=\frac{\bar{y}-\bar{Y}}{\bar{Y}}, e_1=\frac{\bar{x}-\bar{X}}{\bar{X}}, e_2=\frac{s_y^2-S_y^2}{S_y^2}, e_3=\frac{s_x^2-S_x^2}{S_x^2} \text{ such that } \lim_{n\to N}\left|e_h\right|\approx 0,\ h=0,1,2,3$$

$$\left.\begin{array}{l}E\left(e_h\right)=0, h=0,1,2,3., E\left(e_0^2\right)=\gamma C_y^2, E\left(e_1^2\right)=\gamma C_x^2, E\left(e_2^2\right)=\gamma\left(\lambda_{40}-1\right),\\ E\left(e_3^2\right)=\gamma\left(\lambda_{04}-1\right), E\left(e_0e_1\right)=\gamma\rho C_yC_x, E\left(e_0e_2\right)=\gamma C_y\lambda_{30}, E\left(e_0e_3\right)=\gamma C_y\lambda_{12},\\ E\left(e_1e_2\right)=\gamma C_x\lambda_{21}, E\left(e_1e_2\right)=\gamma C_x\lambda_{03}, E\left(e_2e_3\right)=\gamma\left(\lambda_{22}-1\right)\end{array}\right\} \tag{62}$$

Expressing (59), (60) and (61) in terms of error terms, we have:

$$T_{M1}=\left[\begin{array}{l}\dfrac{S_y\left(1+e_2\right)^{1/2}}{2\bar{Y}\left(1+e_0\right)}\left[\dfrac{\bar{X}}{\bar{X}\left(1+e_1\right)}+\dfrac{\bar{X}\left(1+e_1\right)}{\bar{X}}\right]\\ +w_1\left(\bar{X}-\bar{X}\left(1+e_1\right)\right)+w_2\dfrac{S_y\left(1+e_2\right)^{1/2}}{\bar{Y}\left(1+e_0\right)}\end{array}\right]\left[\frac{\bar{X}}{\bar{X}\left(1+e_1\right)}\right] \tag{63}$$

$$T_{M2}=\begin{bmatrix}\dfrac{S_y\left(1+e_2\right)^{1/2}}{2\bar{Y}\left(1+e_0\right)}\left[\dfrac{S_x^2}{S_x^2\left(1+e_3\right)}+\dfrac{S_x^2\left(1+e_3\right)}{S_x^2}\right]\\+w_3\left(S_x^2-S_x^2\left(1+e_3\right)\right)+w_4\dfrac{S_y\left(1+e_2\right)^{1/2}}{\bar{Y}\left(1+e_0\right)}\end{bmatrix}\left[\dfrac{S_x^2}{S_x^2\left(1+e_3\right)}\right] \quad (64)$$

$$T_{M3}=\frac{C_y}{2}\begin{pmatrix}1-e_0\\+e_0^2+\dfrac{e_2}{2}\\-\dfrac{e_0e_2}{2}-\dfrac{e_2^2}{8}\end{pmatrix}\begin{bmatrix}\dfrac{\left(1+e_1\right)}{\left(1+e_3\right)^{1/2}}+\dfrac{\left(1+e_3\right)^{1/2}}{\left(1+e_1\right)}-w_5C_x\begin{pmatrix}e_1-e_1^2-\\\dfrac{e_3}{2}+\dfrac{e_1e_3}{2}+\dfrac{e_3^2}{8}\end{pmatrix}\\+w_6\left(1-e_0+e_0^2+\dfrac{e_2}{2}-\dfrac{e_0e_2}{2}-\dfrac{e_2^2}{8}\right)\end{bmatrix}\begin{bmatrix}1+e_1-\dfrac{e_3}{2}\\-\dfrac{e_1e_2}{2}+\dfrac{3e_3^2}{8}\end{bmatrix} \quad (65)$$

Simplify (63), (64) and (65) up to second degree approximation, we obtained

$$T_{M1}-C_y=-C_y\begin{bmatrix}\begin{pmatrix}e_1-\dfrac{3e_1^2}{2}+e_0-e_0e_1\\-e_0^2-\dfrac{e_2}{2}+\dfrac{e_1e_2}{2}+\dfrac{e_0e_2}{2}+\dfrac{e_2^2}{8}\end{pmatrix}-w_2\begin{pmatrix}1-e_1+e_1^2-e_0+e_0e_1\\+e_0^2+\dfrac{e_2}{2}-\dfrac{e_1e_2}{2}-\dfrac{e_0e_2}{2}-\dfrac{e_2^2}{8}\end{pmatrix}\\+w_1\dfrac{\bar{X}}{C_y}\left(e_1-e_1^2\right)\end{bmatrix} \quad (66)$$

$$T_{M2}-C_y=-C_y\begin{bmatrix}\begin{pmatrix}e_3-\dfrac{3e_3^2}{2}+e_0-e_0e_3\\-e_0^2-\dfrac{e_2}{2}+\dfrac{e_2e_3}{2}+\dfrac{e_0e_2}{2}+\dfrac{e_2^2}{8}\end{pmatrix}+w_3\dfrac{S_x^2}{C_y}\left(e_3-e_3^2\right)\\-w_4\begin{pmatrix}1-e_3+e_3^2-e_0+e_0e_3\\+e_0^2+\dfrac{e_2}{2}-\dfrac{e_2e_3}{2}-\dfrac{e_0e_2}{2}-\dfrac{e_2^2}{8}\end{pmatrix}\end{bmatrix} \quad (67)$$

$$T_{M3}-C_y=C_y\begin{bmatrix}\begin{pmatrix}e_1-\dfrac{e_3}{2}-e_1e_3+\dfrac{e_3^2}{2}+\dfrac{e_1^2}{2}-e_0-e_0e_1\\+\dfrac{e_0e_3}{2}+e_0^2+\dfrac{e_2}{2}+\dfrac{e_1e_2}{2}-\dfrac{e_2e_3}{4}-\dfrac{e_0e_2}{2}-\dfrac{e_0^2}{8}\end{pmatrix}-w_5\dfrac{C_x}{C_y}\left(e_1-\dfrac{e_3}{2}-\dfrac{e_1e_3}{2}+\dfrac{3e_3^2}{8}\right)\\+w_6\left(1+e_1-\dfrac{e_3}{2}-e_0+\dfrac{e_2}{2}-\dfrac{e_1e_3}{2}+\dfrac{3e_3^2}{8}-e_0e_1+\dfrac{e_0e_3}{2}+e_0^2+\dfrac{e_1e_2}{2}-\dfrac{e_2e_3}{4}-\dfrac{e_0e_2}{2}-\dfrac{e_2^2}{8}\right)\end{bmatrix} \quad (68)$$

Taking expectation of (66), (67) and (68) and apply the results of (62) to obtain the bias of the suggested estimators as

$$Bias\left(T_{M1}\right)=-C_y\left[\begin{array}{l}\gamma\left(-\frac{3C_x^2}{2}-\rho C_yC_x-C_y^2+\frac{C_x\lambda_{21}}{2}+\frac{C_y\lambda_{30}}{2}+\frac{\left(\lambda_{40}-1\right)}{8}\right)-w_1\frac{\bar{X}}{C_y}\gamma C_x^2\\+w_2\left(1+\gamma\left(C_x^2+\rho C_yC_x+C_y^2-\frac{C_x\lambda_{21}}{2}-\frac{C_y\lambda_{30}}{2}-\frac{\left(\lambda_{40}-1\right)}{8}\right)\right)\end{array}\right] \tag{69}$$

$$Bias\left(T_{M2}\right)=-C_y\left[\begin{array}{l}\gamma\left(-\frac{3\left(\lambda_{04}-1\right)}{2}-C_y\lambda_{12}-C_y^2+\frac{\left(\lambda_{22}-1\right)}{2}+\frac{C_y\lambda_{30}}{2}+\frac{\left(\lambda_{40}-1\right)}{8}\right)-w_3\frac{S_x^2}{C_y}\gamma\left(\lambda_{04}-1\right)\\+w_4\left(1+\gamma\left(\left(\lambda_{04}-1\right)+C_y\lambda_{12}+C_y^2-\frac{\left(\lambda_{22}-1\right)}{2}-\frac{C_y\lambda_{30}}{2}-\frac{\left(\lambda_{40}-1\right)}{8}\right)\right)\end{array}\right] \tag{70}$$

$$Bias\left(T_{M3}\right)=C_y\left[\begin{array}{l}\gamma\left(\begin{array}{l}-C_x\lambda_{03}-\frac{\left(\lambda_{04}-1\right)}{2}+\frac{C_x^2}{2}-\rho C_yC_x+\frac{C_y\lambda_{12}}{2}\\+C_y^2+\frac{C_x\lambda_{21}}{2}-\frac{\left(\lambda_{22}-1\right)}{4}-\frac{C_y\lambda_{30}}{2}-\frac{\left(\lambda_{40}-1\right)}{8}\end{array}\right)-w_5\delta_2\gamma\left(\begin{array}{l}-\frac{-C_x\lambda_{03}}{2}\\+\frac{3\left(\lambda_{04}-1\right)}{8}\end{array}\right)\\+w_6\left(1-\gamma\left(\begin{array}{l}C_x\lambda_{03}-\frac{3\left(\lambda_{04}-1\right)}{8}+\rho C_yC_x-\frac{C_y\lambda_{12}}{2}-C_y^2\\-\frac{C_x\lambda_{21}}{2}+\frac{\left(\lambda_{22}-1\right)}{4}+\frac{C_y\lambda_{30}}{2}+\frac{\left(\lambda_{40}-1\right)}{8}\end{array}\right)\right)\end{array}\right] \tag{71}$$

Squaring and taking expectation of (67), (68) and (69) and apply the results of (62) to obtain the MSEs of the suggested estimators as

$$MSE\left(T_{M1}\right)=C_y^2\left(A+w_1^2B+w_2^2C+2w_1D-2w_2E-2w_1w_2F\right) \tag{72}$$

$$MSE\left(T_{M2}\right)=C_y^2\left(A_1+w_3^2B_1+w_4^2C_1+2w_3D_1-2w_4E_1-2w_3w_4F_1\right) \tag{73}$$

$$MSE\left(T_{M3}\right)=C_y^2\left(A_2+w_5^2B_2+w_6^2C_2-2w_5D_2+2w_6E_2-2w_5w_6F_2\right) \tag{74}$$

where $A=\gamma\left(C_x^2+C_y^2+2\rho C_yC_x-C_x\lambda_{21}-C_y\lambda_{30}+\frac{\left(\lambda_{40}-1\right)}{4}\right)$, $B=\gamma\delta^2\left(\lambda_{04}-1\right)$, $\delta=\frac{\bar{X}}{C_y}$

$C=1+\gamma\left(3C_x^2+3C_y^2+4\rho C_yC_x-2C_x\lambda_{21}-2C_y\lambda_{30}\right)$, $D=\gamma\delta\left(C_x^2+\rho C_yC_x-\frac{C_x\lambda_{21}}{2}\right)$

$E=\gamma\left(\frac{3C_x\lambda_{21}}{2}-3\rho C_yC_x-\frac{5C_x^2}{2}-2C_y^2+\frac{3C_y\lambda_{30}}{2}-\frac{\left(\lambda_{40}-1\right)}{8}\right)$, $F=\gamma\delta\left(\frac{C_x\lambda_{21}}{2}-\rho C_yC_x-2C_x^2\right)$

$A_1=\gamma\left(\left(\lambda_{04}-1\right)+C_y^2+2C_y\lambda_{12}-\left(\lambda_{22}-1\right)-C_y\lambda_{30}+\frac{\left(\lambda_{40}-1\right)}{4}\right)$, $B_1=\gamma\delta_1^2\left(\lambda_{22}-1\right)$, $\delta_1=\frac{S_x^2}{C_y}$

$C_1=1+\gamma\left(3\left(\lambda_{04}-1\right)+3C_y^2+4C_y\lambda_{12}-2\left(\lambda_{22}-1\right)-2C_y\lambda_{30}\right)$,

$$D_1 = \gamma\delta_1\left(\left(\lambda_{04}-1\right)+C_y\lambda_{12}-\frac{\left(\lambda_{22}-1\right)}{2}\right)$$

$$E_1 = \gamma\left(\frac{3\left(\lambda_{22}-1\right)}{2}-3C_y\lambda_{12}-\frac{5\left(\lambda_{04}-1\right)}{2}-2C_y^2+\frac{3C_y\lambda_{30}}{2}-\frac{\left(\lambda_{40}-1\right)}{8}\right),$$

$$F_1 = \gamma\delta_1\left(\frac{\left(\lambda_{22}-1\right)}{2}-C_y\lambda_{12}-2\left(\lambda_{04}-1\right)\right)$$

$$A_2 = \gamma\left(C_x^2-C_x\lambda_{03}-2\rho C_yC_x+C_x\lambda_{21}+\frac{\left(\lambda_{04}-1\right)}{4}+C_y\lambda_{12}-\frac{\left(\lambda_{22}-1\right)}{2}+C_y^2-C_y\lambda_{30}+\frac{\left(\lambda_{40}-1\right)}{4}\right)$$

$$B_2 = \gamma\left(C_x^2-C_x\lambda_{03}+\frac{\left(\lambda_{04}-1\right)}{4}\right),\ \delta_2=\frac{C_x}{C_y}$$

$$C_2 = 1+\gamma\left(C_x^2-2C_x\lambda_{03}-4\rho C_yC_x+2C_x\lambda_{21}+\left(\lambda_{04}-1\right)+2C_y\lambda_{12}-\left(\lambda_{22}-1\right)+3C_y^2-2C_y\lambda_{30}\right)$$

$$D_2 = \delta_2\gamma\left(C_x^2-C_x\lambda_{03}-\rho C_yC_x+\frac{\left(\lambda_{04}-1\right)}{4}+\frac{C_y\lambda_{12}}{2}+\frac{C_x\lambda_{21}}{2}-\frac{\left(\lambda_{22}-1\right)}{2}\right)$$

$$E_2 = \gamma\left(\frac{3C_x^2}{2}-2C_x\lambda_{03}-3\rho C_yC_x-\frac{3C_x\lambda_{21}}{2}+\frac{3\left(\lambda_{04}-1\right)}{2}+\frac{3C_y\lambda_{12}}{2}-\frac{3\left(\lambda_{22}-1\right)}{4}+2C_y^2-\frac{3C_y\lambda_{30}}{2}+\frac{\left(\lambda_{40}-1\right)}{8}\right)$$

$$F_2 = \delta_2\gamma\left(C_x^2-\frac{3C_x\lambda_{03}}{2}-\rho C_yC_x+\frac{C_x\lambda_{21}}{2}+\frac{5\left(\lambda_{04}-1\right)}{8}+C_y\lambda_{12}+\frac{C_y\lambda_{12}}{2}-\frac{\left(\lambda_{22}-1\right)}{4}\right)$$

Differentiating (72) partially with respect w_1 and w_2, equate to zero and solve for w_1 and w_2 simultaneously, we obtained $w_1=\frac{CD-EF}{F^2-BC}$ and $w_2=\frac{DF-BE}{F^2-BC}$. Substituting the results in (72), we obtained the minimum mean square error of T_{M1} denoted by $MSE\left(T_{M1}\right)_{\min}$;

$$MSE\left(T_{M1}\right)_{\min}=C_y^2\left[A+\frac{\left(CD^2+BE^2-2DEF\right)}{\left(F^2-BC\right)}\right] \quad (75)$$

Differentiating (73) partially with respect w_3 and w_4, equate to zero and solve for w_3 and w_4 simultaneously, we obtained $w_3=\frac{C_1D_1-E_1F_1}{F_1^2-B_1C_1}$ and $w_4=\frac{D_1F_1-B_1E_1}{F_1^2-B_1C_1}$ Substitute the results in (73), we obtained the minimum mean square error of T_{M2} denoted by $MSE\left(T_{M2}\right)_{\min}$

$$MSE\left(T_{M2}\right)_{\min}=C_y^2\left[A_1+\frac{\left(C_1D_1^2+B_1E_1^2-2D_1E_1F_1\right)}{\left(F_1^2-B_1C_1\right)}\right] \quad (76)$$

Similarly, by differentiating (74) partially with respect to w_5 and w_6, equate to zero and solve for w_5 and w_6 simultaneously, we obtained $w_5 = \frac{E_2F_2 - C_2D_2}{F_2^2 - B_2C_2}$ and $w_6 = \frac{B_2E_2 - D_2F_2}{F_2^2 - B_2C_2}$, Substituting the results in (74), we obtained the minimum mean square error of T_{M3} denoted by $MSE(T_{M3})_{\min}$

$$MSE(T_{M3})_{\min} = C_y^2 \left[A_2 + \frac{\left(C_2D_2^{\ 2} + B_2E_2^{\ 2} - 2D_2E_2F_2\right)}{\left(F_2^{\ 2} - B_2C_2\right)} \right] \tag{77}$$

4 Empirical Study

In this section, we carry out an empirical study to elucidate the performance of our proposed estimators with respect to some existing related estimators using two (2) data sets below.

Population 1: [Source: [21], p.399]

X: Area under wheat in 1963, Y: Area under wheat in 1964

$$N = 34, n = 15, \bar{X} = 208.88, \bar{Y} = 199.44, C_x = 0.72, C_y = 0.75, \rho = 0.98, \lambda_{21} = 1.0045, \lambda_{12} = 0.9406,$$
$$\lambda_{40} = 3.6161, \lambda_{04} = 2.8266, \lambda_{30} = 1.1128, \lambda_{03} = 0.9206, \lambda_{22} = 3.0133$$

Population 2: [Source: [22], p.1116]

X: Number of fish caught in year 1993, Y: Number of fish caught in year 1995

$$N = 69, n = 40, \bar{X} = 4591.07, \bar{Y} = 4514.89, C_x = 1.38, C_y = 1.38, \rho = 0.96, \lambda_{21} = 2.19, \lambda_{12} = 2.30,$$
$$\lambda_{40} = 7.66, \lambda_{04} = 9.84, \lambda_{30} = 1.11, \lambda_{03} = 2.52, \lambda_{22} = 8.19$$

Table 1. MSEs and PREs of proposed and other estimators in the study

Estimators	**Population 1**		**Population 2**	
	MSE	**PRE**	**MSE**	**PRE**
$Auxiliary\ Information: \bar{X}, \bar{x}$				
t_0	0.008003575	100.00	0.03808827	100.00
t_{AR}	0.02715658	29.47	0.07645918	49.82
t_1	0.006868341	116.53	0.03731461	102.07
t_2	0.006868341	116.53	0.03731461	102.07
t_3	0.006868341	116.53	0.03731461	102.07
t_4^j	0.006868341	116.53	0.03731461	102.07
t_5^j	0.006868341	116.53	0.03731461	102.07
t_6^j	0.006868341	116.53	0.03731461	102.07

Estimators	Population 1		Population 2	
	MSE	**PRE**	**MSE**	**PRE**
T_{M1}	0.006737495	118.79	0.03404568	111.87
$Auxiliary\ Information: S_X^2, s_x^2$				
t_7^j	0.006962763	114.95	0.037568156	101.38
t_8^j	0.006962763	114.95	0.037568156	101.38
t_9^j	0.006962763	114.95	0.037568156	101.38
t_{10}^j	0.006962763	114.95	0.037568156	101.38
t_{11}^j	0.006962763	114.95	0.037568156	101.38
t_{12}^j	0.006962763	114.95	0.037568156	101.38
T_{M2}	0.006013652	133.09	0.02810758	135.51
$Auxiliary\ Information: C_X = S_X / \bar{X}, c_x = s_x / \bar{x}$				
t_{13}^j	0.001208508	662.27	0.02988236	127.46
t_{14}^j	0.001208508	662.27	0.02988236	127.46
t_{15}^j	0.001208508	662.27	0.02988236	127.46
t_{16}^j	0.001208508	662.27	0.02988236	127.46
t_{17}^j	0.001208508	662.27	0.02988236	127.46
t_{18}^j	0.001208508	662.27	0.02988236	127.46
T_{M3}	0.000787631	1016.16	0.02984082	127.64

5 Results and Discussion

Table 1 shows the MSEs and PREs of the proposed and other estimators considered in the study using information of the two populations 1 and 2. Results obtained from each category revealed that proposed estimators under each category has minimum MSEs and higher PREs compared to other competing existing estimators. These imply that the suggested estimators are more efficient than their counterparts and hve higher chances to produce estimates closer to the true values of means for any population of interest.

6 Conclusions

In our study, we have suggested three difference-cum-ratio estimators for estimating the coefficient of variation of the study variable Y, and these estimators utilized information on population mean, population mean square error and population coefficient of variation of the auxiliary variable X. From the empirical study, the results showed that the proposed estimators were more efficient than the existing estimators considered in the study. Hence we recommend that the proposed estimators should be used in both theoretical and real life applications.

Competing Interests

Authors have declared that no competing interests exist.

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