British Journal of Mathematics & Computer Science
15(6): 1-13, 2016, Article no.BJMCS.25705
ISSN: 2231-0851

SCIENCEDOMAIN *international*
www.sciencedomain.org

Cournot and Stackelberg Equilibria in an Asymmetric Duopoly

Ivan G. Ivanov[1,2*] and Tonya Mateva[2]

[1] *Faculty of Economics and Business Administration, Sofia University St. Kliment Ohridski, Sofia 1113, Bulgaria.*
[2] *College of Dobrich, Shumen University, Shumen, Bulgaria.*

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/BJMCS/2016/25705
Editor(s):
(1) Raducanu Razvan, Department of Applied Mathematics, Al. I. Cuza University, Romania.
Reviewers:
(1) Fang Xiang, University of International Business and Economics, China.
(2) Ijirshar Victor Ushahemba, Benue State University, Makurdi, Nigeria.
(3) Anonymous, Guangdong University of Finance and Economics, China.
Complete Peer review History: http://sciencedomain.org/review-history/14146

***Received:** 16th March 2016*
***Accepted:** 8th April 2016*
***Published:** 13th April 2016*

Original Research Article

Abstract

Aims/Objectives: To study an asymmetric duopoly in gas market where two players produce a homogeneous commodity. One of the players maximizes its profit, while the other- its revenue. The asymmetry also includes a security constraint, saying that the revenue- maximizing player can sell no more than a certain proportion of the quantity of its rival.
Study Design: Interdisciplinary study.
Place and Duration of Study: Department of Information Technologies, College of Dobrich, Shumen University, Bulgaria, between October 2015 and February 2016.
Methodology: We assume that the industry is duopolistic, the product is homogeneous- natural gas and storage is impossible. We impose a security constraint that one of the players cannot produce more than a certain proportion of its rival's quantity and behaves as a revenue maximizer. Cournot and Stackelberg are compared, with and without security constraint.
Results: Results we have reached can be generalized as follows: There is an continuum of Nash-Cournot equilibria and theconstraint is active under some additional conditions; Stackelberg

**Corresponding author: i_ivanov@feb.uni-sofia.bg;*

equilibrium is unique; In both Stackelberg and Cournot model the security constraint punishes player f and rewards player l; Stackelberg game with active security constraint punishes even further player f and the revenue is even lower compared to unconstrained market conditions; As typical to oligopolistic markets price is higher and quantity sold is lower even under imposed security constraint; A special Nash- Cournot equilibrium exists where player f maximizes its revenue. An additional condition should be introduced.

Conclusion: We estimate the constraint's impact on profits, consumer surplus, quantities and price. In addition we offer a special case where one of the player maximizes its revenue and the other earns the highest possible profit.

Keywords: Game theory; gas market; Cournot and Stackelberg equilibria; security constraint.

2010 Mathematics Subject Classification: 49M37; 65K05; 91A40; 91A80.

1 Introduction

Electricity and gas markets have been subject to a significant interest lately and authors have used different approaches for their analysis. Many authors have developed a successive oligopoly approaches to the European electricity and gas markets. Electriciry market analyses are really relevant here because authors use in the analysis of both markets the same approaches and models. The concept of a supply function equilibrium (SFE) has been widely used to investigate both markets. Investigators of both markets have noted how generation capacity constraints may contribute to market power of generation firms.

There are different studies where a supply function is a base in the concept of the supply function equilibrium. It is applied in an electricity market assuming a linear demand function and considering a competitive fringe and several strategic players all having capacity limits and affine marginal costs [1], [2].

Rudkevich in [3] presents a stylized model of the learning process through which power- generating companies could adjust their supply bidding strategies in order to achieve a rational profit-maximizing equilibrium behavior in the form of SFE.

Following Breton and Zaccour [4], we assume that the industry is duopolistic, the product is homogeneous- natural gas and storage is impossible. Genc and Reynolds in [5] have provided conditions under which asymmetric equilibria exist and characterize these equilibria. They have presented a review of the electricity market the equilibrium and non-equilibrium models. The agent and models based supply function has non-equilibrium and equilibrium simulation models under the conditions of both imperfect and perfect competition.

In many papers the focus is on the mixed oligopoly and asymmetric duopoly with a constraint and more specially the capacity constraint. The capacity constraints are very popular in the real word they can be viewed in many economic activities. The effects of the constraint are studied by many researchers (see for example [6], [7],[8],[9] and literature there in). [7] has introduced a model involving public and private firms that produce homogeneous goods by performed capacity constraints. In [8] authors have explored a model of substitutability product under duopoly with capacity constrained input. The main conclusion in the Stackelberg case is that the capacity constraints have important impact and effects on firm size difference and the price difference.

Our investigation and conclusions make up similar studies in the oligopolistic theory, for example [10, 11]. [11] have analyzed the natural gaz market in Spain before the market liberalization and furthermore they have compared the actual data from the National Energy Commission with the theoretical equilibrium predicted by the Stackelberg model. [10] have applied the Stackelberg game model for public input competition. Under this consideration the important conclusions are derived.

We impose a security constraint that one of the players cannot produce more than a certain proportion of its rival's quantity and behaves as a revenue maximizer. Cournot and Stackelberg are compared, with and without security constraint. We estimate the constraint's impact on profits, consumer surplus, quantities and price. In addition we offer a special case where one of the player maximizes its revenue and the other earns the highest possible profit.

2 The Model

We consider a duopoly selling a uniform commodity. Players are labelled respectively l and f ; their corresponding quantities on market are q_l and q_f . Let $Q = q_l + q_f$ denote the total quantity supplied by both market participants and price is represented as the inverse demand function: $p = F(Q)$. We will use a model with linear demand function $D = A - bp$, where A is a constant intercept, though it can be treated as varying over time, or stochastically; $b > 0$ is the demand responsiveness to price. The equilibrium price is determined as a solution of the equation $A - bp = q_l(p) + q_f(p)$, i.e.

$$p = F(Q) = F(q_l + q_f) = \frac{1}{b}(A - q_l - q_f)\,.$$

Player l's production cost is given by:

$$C_l(q_l) = \frac{1}{2}\delta q_l^2 + \omega q_l,$$

where $\delta \geq 0$ and $\omega > 0$ and $2\omega b < A$. Player f's production cost is zero. That is player f is assumed to maximize its revenue. Its costs assumed to be very negligible compared to the profit earned.

The profits of both players are given by :

$$\Pi_l(q_l, Q) = q_l F(Q) - C_l(q_l)\,,$$
$$\Pi_f(q_f, Q) = q_f F(Q)\,.$$

For any reasons, player f is not allowed to supply more than a certain proportion of the player l's quantity, i.e. $q_f \leq \gamma q_l$ and $\gamma \in [\frac{1}{2}, 1]$. We assume that $\gamma \in [\frac{1}{2}, 1]$ in order to limit to some extent the dependence on supplies from player l.

There are some standard [4] assumptions on the cost and demand functions. We verify these classical assumptions for our model:

1. The cost function $C_l(q_l)$ is continuous and nonnegative for $q_l \geq 0$, with $C_l'(q_l) = \delta q_l + \omega > 0$, $C_l''(q_l) = \delta \geq 0$.

2. The inverse demand function $F(Q)$ is defined and continuous with $0 \leq Q \leq A$. $F(Q)$ is twice continuously differentiable on this interval and $F' = -\frac{1}{b} < 0$ and $F'' = 0$.

3. The function $QF(Q) = \frac{Q}{b}(A - Q)$ is strictly concave for $Q \in [0, A]$.

Theorem 2.1. *If $\gamma q_l = q_f$, then the profit $\Pi_l(q_l, Q)$ of the l player is nonnegative if and only if*

$$q_l \in \left[0, \frac{2(A - \omega b)}{2 + 2\gamma + \delta b}\right]\,.$$

Corollary 2.2. *If $\gamma q_l = q_f$, then the function $\Pi_l(q_l, Q)$ has a maximum value*

$$q_l^* = \frac{A - wb}{2 + 2\gamma + \delta b}.$$

The function $\Pi_f(q_l, q_f)$ is nonnegative for $Q = q_l + q_f < A$. This function increases for $q_f \in [0, \frac{A-q_l}{2}]$ and it decreases for $q_f \in [\frac{A-q_l}{2}, A]$. The maximum is reached at the point $q_f = \frac{A-q_l}{2}$.

2.1 The Cournot game

2.1.1 The constrained Cournot game

Both players face the following optimization problems taking into consideration the activeness of the security constraint:

$$\max_{q_l}\ \Pi_l(q_l, q_f) = q_l\,\frac{1}{b}(A - q_l - q_f) - \omega q_l - \frac{1}{2}\delta q_l^2 .$$

$$\max_{q_f}\ \Pi_f(q_l, q_f) = q_f\,\frac{1}{b}(A - q_l - q_f) .$$

The corresponding security constraints for both players are:
$\gamma q_l \geq q_f\,, \quad q_l \geq 0\,, \quad q_f \geq 0\,, \quad \gamma \in [\frac{1}{2}, 1]\,.$

Definition 2.1. The pair (q_l^C, q_f^C) is a Nash-Cournot equilibrium if

$$\Pi_l(q_l, q_f^C) \leq \Pi_l(q_l^C, q_f^C)\,,$$

for all q_l such that $q_l \geq 0,\ \gamma q_l \geq q_l^C$ and

$$\Pi_f(q_l^C, q_f) \leq \Pi_f(q_l^C, q_f^C)\,,$$

for all q_f such that $q_f \geq 0,\ \gamma q_l^C \geq q_f$.

Theorem 2.3. *If*

$$A - 2\omega b > 0 \quad and \quad \gamma < \frac{A\delta b + 2\omega b}{2(A - 2\omega b)}$$

or

$$A - \omega b > 0,\ A - 2\omega b < 0,\ for\ every\ \gamma$$

then the point corresponding to the equilibrium in such a game model is defined by:

$$q_l^C \ \in \ \left[\frac{A-\omega b}{2+\gamma+\delta b},\ \frac{2(A-\omega b)}{2+2\gamma+\delta b}\right]$$

$$q_f^C = \gamma\ q_l^C$$

That is to say that under these conditions there is a continuum of equilibria.

Proof. The necessary and sufficient conditions for Cournot equilibrium have the corresponding form:

$$L_l(q_l, q_f, \lambda_l) = q_l\,\frac{1}{b}(A - q_l - q_f) - \omega q_l - \frac{1}{2}\delta q_l^2 + \lambda_l(\gamma q_l - q_f)$$

$$L_f(q_l, q_f, \lambda_f) = q_f\,\frac{1}{b}(A - q_l - q_f) + \lambda_f(\gamma q_l - q_f)$$

$$L'_{l_{q_l}}(q_l, q_f, \lambda_l) = \frac{1}{b}(A - q_f) - \frac{2}{b}q_l - \omega - \delta q_l + \lambda_l\,\gamma = 0 \tag{2.1}$$

$$\lambda_l \geq 0, \quad \gamma q_l - q_f \geq 0, \quad \lambda_l(\gamma q_l - q_f) = 0 \tag{2.2}$$

$$L'_{f_{q_f}}(q_l, q_f, \lambda_f) = \frac{1}{b}(A - q_l) - \frac{2}{b}q_f - \lambda_f = 0\,, \tag{2.3}$$

$$\lambda_f \geq 0, \quad \gamma q_l - q_f \geq 0, \quad \lambda_f(\gamma q_l - q_f) = 0 \tag{2.4}$$

Assume that there exist a solution (q_l^C, q_f^C) with $q_f^C < \gamma q_l^C$. Then $\lambda_l = \lambda_f = 0$ and using (2.1) and (2.3) we have

$$\gamma q_l^C - q_f^C = \gamma \frac{A - 2b\omega}{3 + 2b\delta} - \frac{A + Ab\delta + b\omega}{3 + 2b\delta} < 0\,,$$

which contradicts conditions (2.2) and (2.4).

Hence $q_f = \gamma q_l$. Considering that $\lambda_l = \lambda_f = 0$ a result has been reached that has the following form:

$$\left| \begin{array}{l} q_l = \frac{A-\omega b}{2+\gamma+\delta b} \\ q_l = \frac{A}{1+2\gamma} \end{array} \right.$$

Taking into consideration that γ is within $\frac{1}{2}$ and 1 it is obvious to see that $\frac{A-\omega b}{2+\gamma+\delta b} < \frac{A}{1+2\gamma}$. This implies a result that every $q_l \in \left[\frac{A-\omega b}{2+\gamma+\delta b},\ \frac{A}{1+2\gamma}\right]$ gives maximum profit for the l-player.

Besides, under the condition that $\gamma q_l = q_f$ the profit of the $l-$ player is nonnegative, i.e. $\Pi_l(q_l, q_f) \geq 0$ if and only if $q_l \in \left[0, \frac{2(A-\omega b)}{2+2\gamma+\delta b}\right]$.

It is easy to show that

$$A - \omega b > 0 \iff \frac{A - \omega b}{2 + \gamma + \delta b} < \frac{2(A - \omega b)}{2 + 2\gamma + \delta b}.$$

Using these results one can verify that there exists an infinity of Nash- Cournot equilibria, given by:

$$q_l^C \in \left[\frac{A-\omega b}{2+\gamma+\delta b},\ \min\left\{\frac{2(A-\omega b)}{2+2\gamma+\delta b},\ \frac{A}{1+2\gamma}\right\}\right],$$
$$q_f^C = \gamma\ q_l^C\,.$$

In order to define the right border of the interval two cases should be considered.

If $A - 2\omega b > 0$, then

$$\frac{2(A - \omega b)}{2 + 2\gamma + \delta b} < \frac{A}{1 + 2\gamma} \iff \gamma < \frac{A\delta b + 2\omega b}{2(A - 2\omega b)}.$$

If $A - 2\omega b < 0$, then

$$\frac{2(A - \omega b)}{2 + 2\gamma + \delta b} < \frac{A}{1 + 2\gamma}\ for\ every\ \gamma > 0.$$

Since

$$q_l^* = \frac{A - wb}{2 + 2\gamma + \delta b} < \frac{A - \omega b}{2 + \gamma + \delta b} = q_l^{C*},$$

then the lower bound q_l^{C*} of the interval maximizes the profit $\left(\Pi_l(q_l^{C*}, \gamma q_l^{C*})\right)$ of the l-player. □

So, for the parameters under the condition $A - \omega b > 0$ in this case of the Cournot game we have

$$q_l^{C*} = \frac{A - \omega b}{2 + \gamma + \delta b}, \qquad q_f^{C*} = \gamma q_l^{C*} = \gamma\ \frac{A - \omega b}{2 + \gamma + \delta b},$$

$$Q^{C*} = q_l^{C*} + q_f^{C*} = \frac{(A - \omega b)(1 + \gamma)}{2 + \gamma + \delta b},$$

$$p^{C*} = \frac{1}{b}(A - Q^{C*}) = \frac{A(1 + \delta b) + \omega b(1 + \gamma)}{b(2 + \gamma + \delta b)},$$

$$\Pi_l^{C*} = \Pi_l(q_l^{C*}, q_f^{C*}) = \frac{(A - \omega b)^2(2 + \delta b)}{2b(2 + \gamma + \delta b)^2},$$

$$\Pi_f^{C*} = \gamma \frac{(A - \omega b)\,[A(1+\delta b) + \omega b(1+\gamma)]}{b(2+\gamma+\delta b)^2}\ .$$

Theorem 2.4. *If*

$$\frac{4\omega b}{1-\delta b} \le A,$$

then the player l must have q_l^{C*} with $\gamma = \hat{\gamma}$. And the player f's corresponding quantity is $\hat{q}_f^{C*}$ for $\hat{\gamma}$. Thus resulting in a special case equilibrium under

$$\hat{Q}^{C*} = q_l^{C*} + \hat{q}_f^{C*} = Q^{C*}(\hat{\gamma}) = \frac{A}{2}, \qquad \hat{P}^{C*} = \frac{1}{b}(A - \hat{Q}^{C*}) = \frac{A}{2b},$$

$$\hat{\Pi}_l^{C*} = \Pi_l^{C*}(\hat{\gamma}) = \frac{(A-2\omega b)^2(2+\delta b)}{8b(1+\delta b)}, \qquad \hat{\Pi}_f^{C*} = \Pi_f^{C*}(\hat{\gamma}) = \frac{A(Ab\delta + 2b\omega)}{4b(1+b\delta)}\ .$$

Proof. By parity of reasoning the maximum revenue for the f player is:

$$\Pi_f(q_f) = q_f\,\frac{1}{b}(A - \frac{1}{\gamma}q_f - q_f)$$

The first condition for the extremum gives

$$q_f^* = \frac{A}{2(\frac{1}{\gamma}+1)} = \frac{A\gamma}{2(\gamma+1)} \ne q_f^{C*}\ .$$

It ensues from this to analyze a special case where the first player chooses to supply q_l^{C*}. In order to ensure its own best possible profit the second player, can choose a quantity $\hat{q}_f$, which corresponds to the next two conditions: $\hat{q}_f = \gamma q_l^{C*}$ and $\hat{q}_f = q_f^*$. Under such circumstances player f will secure a maximum revenue. We find that:

$$\frac{A-\omega b}{2+\gamma+\delta b} = \frac{A}{2(\gamma+1)}$$

$$\gamma = \frac{A\delta b + 2\omega b}{A - 2\omega b} = \hat{\gamma}\,.$$

Since $\gamma \in [\frac{1}{2}, 1]$ a new condition is obtained $\frac{4\omega b}{1-\delta b} \le A$. □

2.1.2 The unconstrained Cournot game

If we exclude the additional security constraint in the Cournot model, the equilibrium point for the two players is given by:

$$q_l^* = \frac{A - 2b\omega}{3+2b\delta} = q_l^{Cu*}, \qquad q_f^* = \frac{A + Ab\delta + b\omega}{3+2b\delta} = q_f^{Cu*}$$

and subsequently

$$Q^{Cu*} = q_l^{Cu*} + q_f^{Cu*} = \frac{2A + Ab\delta - b\omega}{3+2b\delta}, \qquad p^{Cu*} = \frac{A + Ab\delta + b\omega}{b(3+2b\delta)},$$

$$\Pi_l^{Cu*} = \Pi_l(q_l^{Cu*}, q_f^{Cu*}) = \frac{(A-2\omega b)^2\,(2+\delta b)}{2b(3+2b\delta)^2}, \qquad \Pi_f^{Cu*} = \frac{(A + Ab\delta + b\omega)^2}{b(3+2b\delta)^2}\ .$$

2.1.3 Comparison of equilibria in Cournot game

In this section we will compare prices, quantities and profits at the lower bound of the interval in the constrained case, where a unique Nash- Cournot stable equilibrium is reached, together with the unconstrained case. This can occur only if $A-2\omega b>0$ and $\gamma<\frac{A\delta b+2\omega b}{2(A-2\omega b)}$. We use some simple computations based on the results obtained above.

Theorem 2.5. *When $A-2\omega b>0,\ 1-\gamma>0$ and $\gamma<\frac{A\delta b+2\omega b}{2(A-2\omega b)}$ the following results for the market have been reached:*

- *The quantity sold and the profit made by player l are greater in the Nash- Cournot constrained case than in the Nash- Cournot unconstrained case:*

$$q_l^{C*}-q_l^{Cu*}>0\,,\qquad \Pi_l^{C*}-\Pi_l^{Cu*}>0;$$

- *The revenue of player f is greater again in the constrained case, but the quantity sold in that case is less than that in the unconstrained one:*

$$\Pi_f^{C*}-\Pi_f^{Cu*}>0\,,\qquad q_f^{C*}-q_f^{Cu*}<0\,.$$

- *The total market price reached by both players is greater in the constrained case:*

$$p^{C*}-p^{Cu*}>0;$$

- *The total quantity sold to the customers in the constrained case is less than that sold in the Nash- Cournot unconstrained case:*

$$Q^{C*}-Q^{Cu*}<0\,.$$

Proof. Consider the differences

$$q_l^{C*}-q_l^{Cu*}=\frac{A(1-\gamma)+Ab\delta+b\omega+2\omega b\gamma}{(2+\gamma+\delta b)(3+2b\delta)}>0 \iff 1-\gamma>0,$$

$$\Pi_l^{C*}-\Pi_l^{Cu*}=\frac{(2+b\delta)\left(A(1-\gamma)+\omega b+Ab\delta+2\omega v\gamma\right)\left((2b\delta+\gamma+5)(A-2b\omega)+Ab\delta+3b\omega\right)}{2b(2+\gamma+\delta b)^2\,(3+2b\delta)^2}$$

Obviously $\Pi_l^{C*}-\Pi_l^{Cu*}>0$ since $1-\gamma>0$ and $A-2b\omega>0$.

$$q_f^{C*}-q_f^{Cu*}=-(2+\delta b)\frac{A(1-\gamma)+Ab\delta+b\omega+2\gamma\omega b}{(2+\gamma+\delta b)(3+2b\delta)}<0\,.$$

$$\begin{aligned}\Pi_f^{C*}-\Pi_f^{Cu*} &= \gamma\frac{(A-\omega b)\left[A(1+\delta b)+\omega b(1+\gamma)\right]}{b(2+\gamma+\delta b)^2}-\frac{(A+Ab\delta+b\omega)^2}{b(3+2b\delta)^2}\\ &= -\frac{(A+Ab\delta+\omega b-A\gamma+2b\omega\gamma)}{b(2+\gamma+\delta b)^2\,(3+2b\delta)^2}\left(+Ab^3\delta^3+b\omega(5\gamma+4)\right.\\ &\quad \left.+A(4-\gamma)(1+2b\delta)++b^2\omega\delta(4+6\gamma)+b^3\omega\delta^2(1+2\gamma)+Ab^2\delta^2(5-\gamma)\right).\end{aligned}$$

$$p^{C*}-p^{Cu*}=\frac{(1+b\delta)}{b(2+\gamma+\delta b)(3+2b\delta)}\ \left[A(1-\gamma)+b\omega+2\gamma b\omega+Ab\delta\right]>0$$

$$Q^{C*}-Q^{Cu*}=-(1+b\delta)\frac{A(1-\gamma)+b\omega+2\gamma b\omega+Ab\delta}{(2+\gamma+\delta b)(3+2b\delta)}\ <0\,.$$

The theorem is proved. □

2.1.4 Comparison of equilibria in a special case of Cournot game

In this section we will compare prices, quantities and profits in the constrained case, where a unique Nash- Cournot equilibrium is reached and player f maximizes its revenues and player l reaches the maximum possible profit at the bounds of the interval, together with the unconstrained case. This can occur only if $\frac{4\omega b}{1-\delta b} \leq A$. Again we use the same simple computation procedure based on the results obtained above.

Theorem 2.6. *If $\frac{4\omega b}{1-\delta b} \leq A$ and $\gamma = \hat{\gamma}$ the market characteristics are:*

- *The quantity sold and the profit made by player l are greater in the Nash- Cournot constrained case compared to the Nash- Cournot unconstrained case:*
$$q_l^{C*} - q_l^{Cu*} > 0, \qquad \hat{\Pi}_l^{C*} - \Pi_l^{Cu*} > 0\,;$$
- *The revenue of player f is greater again in the constrained case, but the quantity sold in that case is less than that in the unconstrained one:*
$$\hat{\Pi}_f^{C*} - \Pi_f^{Cu*} > 0 \qquad \hat{q}_f^{C*} - q_f^{Cu*} < 0\,;$$
- *The total market price reached by both players is greater in the constrained case:*
$$\hat{p}^{C*} - p^{Cu*} > 0\,;$$
- *The total quantity sold to customers in the constrained case is less than that sold in the Nash- Cournot unconstrained case:*
$$\hat{Q}^{C*} - Q^{Cu*} < 0\,.$$

Proof. This theorem follows immediately from Theorem 2.5 using $\gamma = \hat{\gamma}$. □

2.2 The Stackelberg game

Now we consider Stackelberg game model applied for natural gas market. Players again are two : a leader (the player l) and a follower (the player f).

Definition 2.2. The pair (q_l^S, q_f^S) is a Stackalberg equilibrium if
$$\Pi_l(q_l, q_f^S(q_l)) \leq \Pi_l(q_l^S, q_f^S(q_l))\,,$$
for all q_l such that $q_l \geq 0$ and $q_f^S(q_l)$ is defined by the reaction function of the follower.

Profit maximization functions of both firms are the same. Studying the Stackelberg model needs attention to two cases when the security constraint is active and not active. They depend on demand elasticity. Breton and Zaccour formulate these cases as [4]:

a) the case of active constraint $(q_f = \gamma q_l)$,
b) the case of active constraint $(q_f = \gamma q_l)$.

Follow [[4]] the constraint is active when
$$-E(Q) \geq \frac{\gamma}{1+\gamma}, \quad \text{for all} \quad Q \in (0, A] \tag{2.5}$$
and the constraint is not active
$$-E(Q) < \frac{\gamma}{1+\gamma}, \quad \text{for all} \quad Q \in (0, A], \tag{2.6}$$
where
$$-E(Q) = \frac{1}{F'(Q)}\frac{F(Q)}{Q} = \frac{A}{Q} - 1.$$

2.2.1 The constrained Stackelberg game

Under the assumption of the security constraint the players face the following optimization problems in Stackelberg model:

$$\max_{q_l} \; \Pi_l(q_l, q_f) = q_l \, \frac{1}{b}(A - q_l - q_f(q_l)) - \omega q_l - \frac{1}{2}\delta q_l^2 \, ,$$

$$\max_{q_f} \; \Pi_f(q_l, q_f) = q_f \, \frac{1}{b}(A - q_l - q_f)$$

where $\gamma q_l \geq q_f \, , \quad q_l \geq 0 \, , \quad q_f \geq 0 \, , \quad \gamma \in [\frac{1}{2}, 1] \, .$

The first order condition of the Lagrange function leads to formulating these requirements

$$q_f = \frac{1}{2}(A - q_l - \lambda b), \quad \lambda \geq 0, \quad \gamma q_l \geq q_f, \quad \lambda(\gamma q_l - q_f) = 0.$$

The result is

$$q_l = \frac{A + \lambda b - 2\omega b}{2(1 + \delta b)}$$

and therefore we find the optimal quantity q_f

$$q_f = \frac{A - \lambda b}{2} - \frac{1}{2} q_l = \frac{A - 3\lambda b + 2Ab\delta - 2\lambda\delta b^2 + 2\omega b}{4(1 + \delta)} \, .$$

Theorem 2.7. *Under Stackelberg game there exist two cases depending on type of the constraint. If* $\gamma \leq \frac{A + 2Ab\delta + 2\omega b}{2(A - 2\omega b)}$ *the constraint is active. In other cases it is not active. In both cases there exists a unique Stackelberg equilibrium.*

Proof. To prove this statement we will analyze two different cases, where the security constraint is active and not active. Stackelberg model with active security constraint ($\gamma q_l = q_f$) results are as follows:

$$\lambda = \frac{A + 4\gamma\omega b + 2Ab\delta + 2\omega b - 2A\gamma}{b(3 + 2b\delta + 2\gamma)} \, ,$$

and

$$q_l = \frac{2(A - \omega b)}{3 + 2\delta b + 2\gamma} \, , \qquad q_f = \frac{2\gamma(A - \omega b)}{3 + 2\delta b + 2\gamma} \, .$$

We have

$$Q = \frac{2(1 + \gamma)\,(A - \omega b)}{3 + 2\delta b + 2\gamma}$$

and then

$$\frac{A}{Q} - 1 = \frac{A + 2Ab\delta + 2\omega b + 2\gamma\omega b}{2(A - \omega b)(1 + \gamma)} \, .$$

From condition (2.5) we obtain

$$\gamma \leq \frac{A + 2Ab\delta + 2\omega b}{2(A - 2\omega b)} = \gamma_a \, .$$

Note that in this case ($\gamma < \gamma_a$) we have $\lambda > 0$.

The same model, but with **no security constraint** gives these results. Obviously, if

$$\gamma > \gamma_a$$

the constraint will be inactive ($\gamma q_l > q_f$).

We have $\lambda = 0$ and then

$$q_f = \frac{A - q_l}{2}, \quad q_l = \frac{A - 2\omega b}{2(1+\delta b)}$$

or

$$q_f = \frac{A + 2A\delta b + 2\omega b}{4(1+\delta b)}\,.$$

In this case for $\gamma q_l - q_f$ we have

$$\gamma q_l - q_f = \frac{2A\gamma - 4\gamma\omega b - A - 2A\delta b - 2\omega b}{4(1+\delta b)}$$

and

$$\gamma q_l - q_f > 0 \iff \gamma > \gamma_a\,.$$

□

The total market values under active security constraint ($\lambda > 0$) are:

$$q_l^{Sc} = \frac{2(A-\omega b)}{3 + 2\delta b + 2\gamma}, \qquad q_f^{Sc} = \frac{2\gamma(A-\omega b)}{3 + 2\delta b + 2\gamma}\,,$$

$$\lambda = \frac{A + 4\gamma\omega b + 2Ab\delta + 2\omega b - 2A\gamma}{b(3 + 2b\delta + 2\gamma)} > 0\,,$$

$$Q^c = \frac{2(1+\gamma)\,(A-\omega b)}{3 + 2\delta b + 2\gamma}\,, \qquad p^c = \frac{A(1+2\delta b) + 2b\omega(1+\gamma)}{b(3 + 2\delta b + 2\gamma)}\,,$$

$$\Pi_l^{Sc} = \frac{2(A - b\omega)^2(1+\delta b)}{b(3 + 2\gamma + 2\delta b)^2}\,,$$

$$\Pi_f^{Sc} = \frac{2\gamma(A - b\omega)\,(A(1+2\delta b) + 2b\omega(1+\gamma))}{b(3 + 2\gamma + 2\delta b)^2}\,.$$

2.2.2 The unconstrained Stackelberg game

We study the problem for maximizing the profit of both players without a security constraint:

$$\begin{aligned} \Pi_l &= q_l\,\frac{1}{b}\,(A - q_l - q_f - \omega b) - \frac{1}{2}\delta q_l^2\,, \\ \Pi_f &= q_f\,\frac{1}{b}\,(A - q_l - q_f). \end{aligned}$$

Using the first order condition for maximizing the profit we find the leader's and the follower's corresponding quantities:

$$q_l = \frac{A - 2\omega b}{2(1+\delta b)},$$

$$q_f = \frac{A - q_l}{2} = \frac{A + 2A\delta b + 2\omega b}{4(1+\delta b)}\,.$$

The values of the quantities sold by both players, their profits, the market price and quantity in the unconstrained Stackelberg game are as follows:

$$q_l^{Su} = \frac{A - 2\omega b}{2(1+\delta b)}\,, \qquad q_f^{Su} = \frac{A + 2A\delta b + 2\omega b}{4(1+\delta b)},$$

$$Q^u = \frac{3A + 2A\delta b - 2b\omega}{4(1+\delta b)} \qquad p^u = \frac{A + 2A\delta b + 2b\omega}{4b(1+\delta b)},$$

$$\Pi_l^{Su} = \frac{(A - 2b\omega)^2}{8(1+\delta b)}\,, \qquad \Pi_f^{Su} = \frac{(A + 2A\delta b + 2b\omega)^2}{16b(1+\delta b)^2}\,.$$

2.2.3 Comparison of Stackelberg equilibria

We will use the above calculated results for the market in the constrained and unconstrained Stackelberg game to make some simple calculations.

Theorem 2.8. *Assuming that*

$$\frac{A + 2A\delta b + 2b\omega}{2(A - 2b\omega)} > \gamma$$

we reach the following results for the comparison:

- *The quantity sold by player l is greater in the constrained case. But on the other hand player f faces a smaller quantity sold*

$$q_l^{Sc} - q_l^{Su} > 0\,, \qquad q_f^{Sc} - q_f^{Su} < 0\,;$$

- *The total market price is greater in the constrained case.*

$$p^c - p^u > 0\,;$$

- *The quantity sold to all customers is less in the constrained Stackelberg case than that in the unconstrained one.*

$$Q^c - Q^u < 0\,;$$

- *The profit of the leader is greater in the constrained case and the profit of the follower is less in the same case.*

$$\Pi_l^{Sc} - \Pi_l^{Su} \;>\; 0\,, \quad \Pi_f^{Sc} - \Pi_f^{Su} \;<\; 0\,.$$

Proof. We have

$$q_l^{Sc} - q_l^{Su} = \frac{2(A - \omega b)}{3 + 2\delta b + 2\gamma} - \frac{A - 2\omega b}{2(1 + \delta b)} = \frac{A + 2A\delta b + 2b\omega - 2\gamma(A - 2b\omega)}{2(1 + \delta b)(3 + 2\gamma + 2\delta b)} > 0$$

since

$$\frac{A + 2A\delta b + 2b\omega}{2(A - 2b\omega)} > \gamma\,.$$

$$q_f^{Sc} - q_f^{Su} = \frac{(3 + 2\delta b)(2\gamma(A - 2b\omega) - (A + 2A\delta b + 2b\omega))}{2(1 + \delta b)(4(1 + \delta b)(3 + 2\gamma + 2\delta b)} < 0\,.$$

$$\begin{aligned} p^c - p^u \quad &= \quad \frac{A(1 + 2\delta b) + 2b\omega(1 + \gamma)}{b(3 + 2\delta b + 2\gamma)} - \frac{A + 2A\delta b + 2b\omega}{4b(1 + \delta b)} \\ &= \quad \frac{(1 + 2\delta b)\,(A + 2A\delta b + 2b\omega - 2\gamma(A - 2b\omega))}{4b(1 + \delta b)(3 + 2\gamma + 2\delta b)} > 0\,. \end{aligned}$$

$$Q^c - Q^u = \frac{(1 + 2\delta b)^2(2\gamma(A - 2b\omega) - (A + 2A\delta b + 2b\omega))}{4(1 + \delta b)(3 + 2\gamma + 2\delta b)} < 0\,.$$

Compare the profits of the leader

$$\Pi_l^{Sc} - \Pi_l^{Su} = \frac{2(A - b\omega)^2(1 + \delta b)}{b(3 + 2\gamma + 2\delta b)^2} - \frac{(A - 2b\omega)^2}{8(1 + \delta b)} =$$

$$= \frac{(A + 2b\omega + 2Ab\delta - 2A\gamma + 4\omega\gamma b)(7A + 6Ab\delta + 2A\gamma - 2b\omega(5 + 4b\delta + 2\gamma))}{8b(1 + \delta b)(3 + 2\gamma + 2\delta b)^2} \;>\; 0$$

since

$$7A + 6Ab\delta + 2A\gamma - 2b\omega(5 + 4b\delta + 2\gamma) =$$

$$= (A - 2b\omega)(3 + 5b\omega + 2\gamma) + 4A + b\omega + Ab\delta > 0\,.$$

Compare the profits of the follower

$$\begin{aligned}
\Pi_f^{Sc} - \Pi_f^{Su} &= \frac{2\gamma(A - b\omega)\left(A(1+2\delta b) + 2b\omega(1+\gamma)\right)}{b(3+2\gamma+2\delta b)^2} - \frac{(A + 2A\delta b + 2b\omega)^2}{16b(1+\delta b)^2} \\
&= -\frac{1}{16}\,\frac{2\omega b + 4\omega b\gamma + A - 2\gamma A + 2Ab\delta}{b(3+2\gamma+2\delta b)^2\,(1+\delta b)^2} \times \\
&\quad \times \left(8\omega b^3\delta^2 + 7A + 2A(1-\gamma) + 32\omega b^2\gamma\delta \right. \\
&\quad + 16\omega b^3\gamma\delta^2 + 8Ab^3\delta^3 + 18\omega b + 22Ab\delta + 8Ab\delta(1-\gamma) \\
&\quad + \left. 20\omega b\gamma + 24\omega b^2\delta + 20A\delta^2 b^2 + 8A\delta^2 b^2(1-\gamma)\right) < 0\,.
\end{aligned}$$

□

Studying the Stackelberg game we infer the conclusions that the Stackelberg equilibrium is unique in both cases which depending on type of the constraint and the profit of the leader is greater in the constrained case and the profit of the follower is less in the same case. Moreover, our conclusions correspond and confirm the findings in [8].

3 Conclusions

Results we have reached can be generalized as follows:

- There is an continuum of Nash- Cournot equilibria and the constraint is active under some additional conditions;
- Stackelberg equilibrium is unique;
- In both Stackelberg and Cournot model the security constraint punishes player f and rewards player l;
- Stackelberg game with active security constraint punishes even further player f and the revenue is even lower compared to unconstrained market conditions;
- As typical to oligopolistic markets price is higher and quantity sold is lower even under imposed security constraint;
- A special Nash- Cournot equilibrium exists where player f maximizes its revenue. An additional condition should be introduced.

Once we have made these conclusions we can return to the European gas market which we are actually analyzing. As it was already said, for the economic growth to continue as it is appreciated European countries need a continuous flow of gas supplies. Russia's gas imports are increasing over the last years. It can be seen from the results we have reached that the introduction of such a constraint can improve the reliability of gas supply from Russia. It is beneficial for Russia because in both Stackelberg and Cournot model the constraint rewards her and gives Russia higher profit. Russia gets a higher price than the case without constraint and has a better incentive to invest in new projects. In the constrained Cournot model the revenue of the second player is also greater than that in the unconstrained one. Taking into consideration the development of new transit corridors from Russia to Europe,that shall reduce Russia's costs [12],[13] and the fact that from the start of gas deliveries to Europe the Russians have maintained a remarkable stability of supply, we can suppose that security-of-supply will not be threatened. But it seems that all this is at the expense of the consumers who pay higher prices.

Acknowledgement

The present research paper was supported in a part by the EEA Scholarship Programme BG09 Project Grant D03-91 under the European Economic Area Financial Mechanism.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Baldick R, Grant R, Kahn E. Theory and application of linear supply function equilibrium in electricity markets. Journal of Regulatory Economics. 2004;25(2):143-167.

[2] Anderson EJ, Hu X. Asymmetric supply function equilibria with forward contracts. Journal of Optimization Theory and Applications. 2012;152(1):198-224.

[3] Rudkevich A. Supply function equilibrium: Theory and applications. Proceedings of the 36th Hawaii International Conference on System Science; 2002.

[4] Breton M, Zaccour G. Equilibria in an asymmetric duopoly facing a security constraint. Energy Economics. 2001;23:457-475.

[5] Genc T, Reynolds S. Supply function equilibria with capacity constraints and pivotal suppliers. International Journal of Industrial Organization. 2011;29(4):432-442.

[6] Eso P, Nocke V, White L. Competition for scarce resource. RAND Journal of Economics. 2010;41(3):524-548.

[7] Nie PY. Effects of capacity constraints on mixed duopoly. Journal of Economics. 2014;112(3): 283-294.

[8] Nie PY, Chen YH. Duopoly competitions with capacity constrained input. Economic Modelling. 2012;29(5):1715-1721.

[9] Nie PY, Yang YC. Cost-reduction innovation under mixed economy. Economics, Discussion, Papers, 2015, No 2015-68, Kiel Institute for the World Economy.
Available: http://www.economics-ejournal.org/economics/discussionpapers/2015-68

[10] Liu Y, MartinezVazquez J. Public input competition under stackelberg equilibrium: A note. international center for public policy. Andrew Young School of Policy Studies, 2014, Working Paper 14-02.

[11] Miguel JMS, Bergillos YH. Natural Gas market in Spain before market liberalization. Universidad de Sevilla. Spain; 2013. Available:
https://www.hse.ru/data/2013/09/18/1275626811/Miguel%20Jesus_TFM-Ingles-III.pdf

[12] Hirschhausen C, Meinhart B, Ferdinand P. Transporting russian gas to western Europe-a simulation analysis. The Energy Journal. 2005;26(2):49-68.

[13] Chyong CK, Noel P, Reiner DM. The economics of the nord stream pipeline system. Cambridge Working Paper in Economics; 2010.
Available: http://www.econ.cam.ac.uk/dae/repec/cam/pdf/cwpe1051.pdf

---

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://sciencedomain.org/review-history/14146